

# Adverse selection in the labour market and the demand for vocational education

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## Abstract

In this paper, I show that displacement of high-school workers from routine jobs can be understood as the labour-market response to an adverse selection problem. The adverse selection problem arises because employment contracts do not systematically discriminate against education, even though over-qualified workers are relatively more likely to quit routine jobs. The labour market equilibrium distorts the labour market outcomes of high school graduates by inefficiently increasing their unemployment rate, in order to separate them from overqualified college graduates. In addition, the labour market response to the adverse selection problem creates a demand for post-secondary vocational education, which is valuable because it acts as an entry barrier that prevents college graduates from using routine jobs as stepping-stones towards better jobs.

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# 1 Introduction

There is a common perception that high school graduates are being displaced from jobs involving mainly routine tasks. These jobs are increasingly automated, offshored, or performed by over-qualified workers, that is, workers whose qualifications exceed job requirements. The roles of skill-biased technological change and globalization in the determination of labour market conditions faced by less educated workers are widely recognized,<sup>1</sup> but the role of over-qualification remains unclear. Many studies assume that educated and uneducated workers compete for the same scarce jobs.<sup>2</sup> Yet, firm-level data does not support the view that over-qualified workers directly crowd out less educated workers.<sup>3</sup> Moreover, over-qualified workers are likely to use their current job as a stepping stone towards better jobs. But if over-qualified workers have a relatively higher quit rate, it is unclear why they should be the ones displacing other workers, especially from routine jobs, where differences in productivity are not likely to be very large.

In this paper, I show that displacement of high-school graduates from employment can be understood as the labour-market response to an adverse selection problem. The adverse selection problem arises because employment contracts do not systematically discriminate against education, even though over-qualified workers are relatively more likely to quit those routine jobs. In addition, college graduates are less willing to wait for a routine job than high school workers are. Consequently, the labour market equilibrium generates inefficient unemployment of high school graduates in order to separate them from overqualified college graduates. Moreover, I argue that this mechanism explains why vocational education has a higher market value than it is commonly thought.

Most studies of the role of education in the labour market focus on high school and college workers. Yet, the increasing role of vocational education, which is the post-secondary education

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<sup>1</sup>See Autor, Levy, and Murnane (2003) on routine jobs, Goos, Manning, and Salomons (2014) on skill biased technological change versus globalization and Autor, Dorn, Hanson, and Song (2014) on exposure to international trade.

<sup>2</sup>See Gautier (2002), Albrecht and Vroman (2002), Dolado, Jansen, and Jimeno (2009), Acemoglu and Autor (2011), Beaudry, Green, and Sand (2013) and Barnichon and Zylberberg (2014).

<sup>3</sup>Gautier, Van den Berg, Van Ours, and Ridder (2002) find no evidence at the firm-level that firms upgrade their work force during low employment years.

that focuses primarily on providing occupationally specific preparation, is striking. In 2010, for example, about 1,650,000 bachelor degrees and about 1,442,000 vocational education and training (VET) credentials were awarded in the U.S. VET credentials awarded per year increased by 60 percent from 2000 to 2010, growing much faster than the 34 percent increase of bachelor degrees. In this paper I argue that an important part of the labour-market value of vocational education stems from the fact that it acts as an entry barrier to exclude overqualified college graduates from some routine jobs. In this context, skill-biased technological change can explain the rise of vocational education over the past decades.

I begin by analyzing a labour market model with two types of workers and two types of jobs, and search frictions. I have in mind jobs that involve mainly cognitive tasks, but some of those jobs involve mainly routine tasks and some involve mainly non-routine tasks. Workers are assumed to have different educational backgrounds and are referred to as high school and college workers. For simplicity, I assume that all workers can perform routine tasks, and that high school and college workers are equally productive in these tasks. However, only college workers can perform non-routine tasks. College workers employed in routine jobs can search for non-routine jobs while on-the-job.

In this context, routine jobs have value for college workers as a stepping stone toward non-routine jobs. Employers offering these jobs understand that college workers are relatively less profitable than high school workers, because their quit rate is higher. However, employment contracts cannot discriminate effectively against education — either because such a form of discrimination is illegal or because college graduates cannot be compelled to disclose their college degree. This creates an adverse selection problem: employers seeking to fill routine jobs would prefer to hire high school graduates, but college graduates have incentives mimic the search behaviour of high school graduates.

I show that any competitive search equilibrium of the model separates college and high school graduates searching for routine jobs into different markets.<sup>4</sup> The separation, however, is at the cost

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<sup>4</sup>I extend the work of Guerrieri, Shimer, and Wright (2010) on competitive search equilibrium with adverse

of generating inefficient unemployment for high school graduates. This is because college workers are less willing to wait for a routine job than high school workers are. College workers search for routine jobs to transition into better jobs, so they are willing to accept a relatively lower wage as long as the job-finding rate is sufficiently high. In contrast, high school graduates are more likely to wait longer for a relatively higher wage, since they have a longer expected job tenure. In equilibrium, jobs that attract high school graduates offer a relatively higher wage,<sup>5</sup> and a corresponding inefficiently low job-arrival rate to discourage educated workers from applying. As a result, high school workers are displaced from employment by over-qualified college workers.

In this context, I argue that post-secondary vocational education has a higher market value than is commonly thought. The displacement effect through adverse selection is the key to understanding the value of vocational education as an entry barrier. Besides the obvious benefit of skill development, vocational education acts as an entry barrier because jobs requiring vocational education can exclude over-qualified workers from seeking stepping-stone jobs. By extending the baseline model to include educational choices, I show that the adverse selection problem creates a demand for vocational education. Since employment contracts cannot discriminate against workers with more education, but can condition on required training, vocational education successfully excludes college workers from seeking stepping-stone jobs. As a result, employers are able to offer contracts that do not suffer from adverse selection. I argue that introducing costly vocational education into the labour market makes workers ex-ante better off. Vocational education helps screen workers who are more committed to the occupation and improves labour market efficiency. As in the traditional signalling model (Spence, 1973), where college education helps employers to screen workers with high ability, vocational education acts as an institutional solution to adverse selection and helps employers screen the desired type of workers, in this case the high school graduates.

Occupational licensing is an extreme example of entry barriers as it is often viewed to have no impact on productivity. According to Kleiner and Krueger (2013), in 2008 nearly 30 percent of the workers with more than high school education, but not a bachelor degree, were required to hold

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selection to study a dynamic labour market equilibrium that includes on-the-job search.

<sup>5</sup>Wage here represents the total labour income from the job that includes for example benefits and pensions. Jobs with high turnover rate are often less generous in these benefits.

a license. Many popular fields of vocational education such as health care and “trades” in areas such as manufacturing, construction, repair, and transportation, also prepare students to obtain occupational licensing. One concern is that, as an entry barrier, occupational licensing may cause job losses by increasing employment costs. While this concern is widespread in policy papers (see Kleiner (2005)) and in the media, my results imply that occupational licensing can also have an important benefit as an entry barrier. Imposing restrictions on entry provides protection for low to middle-skilled workers from the competition of college graduates seeking for stepping-stone jobs. Entry barriers such as vocational education and occupational licensing can mitigate the distortion caused by adverse selection by screening out workers who are not pursuing a career in the particular occupation. This is in sharp contrast to standard competitive market models, where a barrier to entry creates monopoly power and is unambiguously welfare reducing.

Finally, I relate the outcomes of this model to the increase in post-secondary educational attainment over the past few decades. Autor et al. (2010) point out that one puzzle in the U.S. labour market is that the relative supply of college-educated workers is not growing fast enough, given the steep rise in the college-versus-high-school earnings ratio. This paper suggests that much of the increase in post-secondary education may take the form of non-bachelor post-secondary vocational education. The reason for this is that skill-biased technological change has not only increased the return to college, relative to high school, but also the return to vocational education. I simulate the model to examine the effect of a skill-biased technological change that is complementary to non-routine jobs but a substitute to routine jobs, combined with an increase in university tuition fees to replicate the distribution of education levels of 2010. I find that while the increasing productivity of non-routine jobs leads to a direct increase in the educational attainment at the university level, it also lowers the return to high school by further displacing high school workers from employment. Therefore such a change also increases the incentive for high school workers to attain both college and vocational education. In addition, the reduction in the productivity of some routine jobs has worsened the labour market outcome of high school workers, leading to the increased popularity of VET. Overall, together with an increase in university tuition, skill-biased technological change can explain the increase of both college degrees and post-secondary VET certificates over the past thirty years.

The theory I propose is in contrast with previous work that emphasizes the displacement or “crowding-out” of less-educated workers by high skilled-workers. In the frictionless competitive models of Acemoglu and Autor (2011) and Beaudry, Green, and Sand (2013), no externality is generated by employing high-skilled workers in less-skilled jobs, since replacing less-educated workers by high-skilled ones is an efficient market adjustment. In models with random matching, such as Gautier (2002), Albrecht and Vroman (2002) and Dolado, Jansen, and Jimeno (2009), search externalities rely crucially on the assumption that high-skilled workers and less-educated workers are matched randomly with the same routine employers. What remains unexplained is whether this displacement exists when workers have incentives to apply to different employers. I show that over-qualified workers create a negative spillover effect on low-skilled workers even when they search for different routine jobs. Allowing workers to direct their search is crucial for understanding the mechanism that underlies the displacement of high school graduates and its policy implications.

The rest of the paper is organized as follows. Section 2 lays out the model. Section 3 characterizes the equilibrium with the displacement effect. Section 4 extends the model with educational choices and discusses the implication on vocational education. I also present a numerical simulation to discuss the implications of skill biased technological changes. Section 5 concludes.

## 2 Model

### 2.1 Agents and Markets

Time is discrete and the horizon is infinite. All agents are risk neutral and discount the future at a rate  $r > 0$ . There are two types of workers, distinguished by their observable education background  $i \in \{1, 2\}$ , where type 1 represents workers with a high school diploma only while type 2 workers can be interpreted as workers with a college/university degree. The measure of workers is normalized to 1, with a fixed fraction  $\pi$  of college workers. There are also two types of jobs, job type  $j \in \{1, 2\}$ . Type-2 jobs represent college jobs which involve mostly non-routine cognitive tasks and require a college/bachelor degree while type-1 jobs represent non-college routine jobs for which the skill requirements are met by all workers. Each job requires only one worker to operate. The

productivity of a match is described in Table 1. A non-college worker can only perform non-college jobs while a college worker can perform both jobs and is more productive when matched with a type-2 job than with a type-1 job. For simplicity, I assume that college and non-college workers are equally productive in a non-college job. The main results of the paper hold both if college workers are less or more productive than high school workers in non-college jobs. The distribution of jobs is determined endogenously by free entry.

**Table 1:** Match Productivity

Worker	Job	
	Non-college	College
High School	$y_1$	0
College	$y_1$	$y_2 > y_1$

There are four stages within each period: production, separation, search and matching. In the beginning of a period, all workers are either employed or unemployed. In the production stage, all matches produce and employed workers collect wages while unemployed workers receive benefit  $b < y_1$ , which represents unemployment benefits or the value of home production. In the separation stage, an exogenous separation shock occurs with probability  $\delta > 0$ . In this case, the job is destroyed, while the worker becomes unemployed and starts job-searching in the next period. In the search stage, employed workers and those who were unemployed at the beginning of the period can search while employers post job vacancies. In the matching stage, workers and firms are brought into contact to form new matches. If an employed worker receives an outside offer, her current employer can choose to make a take-it-or-leave-it counter-offer. The worker then chooses whether to accept the poaching offer and switch to the poaching employer or reject it and stay with the incumbent employer.

To understand the impact of over-education on displacing high school workers, the equilibrium requires some fractions of college workers applying to non-college jobs. As the focus of this paper is about the labour market performance of high school workers rather than college workers, over-education is treated in a reduced-form way. I assume that when searching for college jobs, an

unemployed college worker faces a cost  $c > 0$ .<sup>6</sup> The cost is a random draw from a continuous distribution with cumulative distribution function  $F(\cdot)$  and is i.i.d. across workers and over time. It is a relative cost and reflects the relative difficulty of applying for a college versus a non-college job for a college worker. The cost is revealed before the search stage and a college worker determines where to search based on the realization of the cost. For simplicity, I assume no cost is incurred when workers search on job, and none of my results rely on this assumption.

The labour market state of a worker includes whether the worker is employed or unemployed, as well as the wage and employer type if the worker is employed. Given a wage  $\omega \in R^+$  and a job type  $j \in \{0, 1, 2\}$  (with 0 being absent of an employer, i.e. unemployment), let  $\ell = (\omega, j)$  denote the labour market state of an employed worker and  $\ell = (b, 0)$  the labour market state of an unemployed worker. The (full) state of a worker includes her labour market state and the educational background, which is the type of the worker. The state of a worker is perfectly observable.

Since the state of a worker is observable, firms can post menus that offer different contracts conditional on the state of a worker. With free entry, however, posting a menu of contracts is equivalent to separating workers in different markets. This is because no menu in equilibrium can attract workers from more than one state while making positive profits with some workers and non-positive profits with others. The workers who generate positive profits for the employers have incentives to deviate to a menu that makes zero profits from all states of workers to the employers. Instead, all firms post contracts that either attract only one state of workers or attract more than one state of workers but make zero profits from all workers. This is equivalent to a labour market where each firm posts one contract at separated markets for workers with different states. Without loss of generality, I focus on a labour market where each contract is offered to one state of worker in the rest of the paper. The contracting environment is that employers commit to a wage that remains constant until an outside offer is received by the employee. Once an outside offer is received, employers can respond by offering a counter-offer. The renegotiation of the wage is based on mutual agreement.

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<sup>6</sup>See Chen, Doyle, and Gonzalez (2015) for modelling endogenous over-education and Albrecht and Vroman (2002), Gautier (2002) for modelling of over-education under random matching.



The key assumption of the model is that employers offering non-college jobs cannot post contracts that exclude college workers from applying. While vacancy posts for college jobs can have skill requirements, contracts that are posted by employers with non-college jobs are restricted from discriminating against overeducated workers. The reason that an employer of a non-college job might want to separate college workers from high school workers is because workers with different educational background have different career paths. Unlike high school workers, college workers are more likely to quit a non-college job because they might take non-college jobs as a stepping-stone to college jobs. This non-discrimination condition is consistent with the labour market observation that while educational requirements are often observed in job advertisements, job posts that restrict workers with more education from applying are rare. Moreover, restricting more-educated workers is difficult to implement. One reason is that hiding part of one's educational background and omitting credentials is less costly than falsifying credentials. Another reason is that even if some employers commit not to hire more educated workers ex-ante, such commitment is not credible ex-post. With frictional search and job posting costs, it may not be profitable for the employer to reject a more-educated applicant once she shows up. Therefore, although the educational background of a worker is observable to employers, the employer of a non-college job cannot exclude college workers by contract restriction. As a result, even though employers can post contracts to workers of a specific state, a non-college job employers may face both college and non-college applicants.

The structure of the labour market is as follows. Each period a continuum of markets may open. A market is characterized by an employment contract  $x$  and a market queue length  $q$ , which is equal to the ratio of workers to employers in that particular market. A market is indexed by the contract  $x$ . A contract  $x = \{(\omega, j), \ell\} \in X$  specifies the job offer  $(\omega, j)$  which includes a wage  $\omega$  and the job type  $j$  on the employer, and the labour market status of workers  $\ell$  that the contract is made to. While workers with different status do not compete directly with each other, workers of different types may apply for the same contract. The queue mapping  $Q : X \rightarrow \mathbb{R}^+$  maps each market into a queue length. Search is directed as opposed to random matching. Each employer can post any feasible contract with a flow cost  $k_j > 0$  for job type  $j$  and each worker can direct her

search to any market. Both employers and workers take into account the trade-off that the higher is the wage, the longer is the queue length.

Workers and firms in the same market are brought into contact by a matching technology in the matching stage. Matching is bilateral, so each worker meets at most one employer and vice versa. Workers who search in a market  $x$  with market queue length  $q$  meet an employer with probability  $f(q)$  and employers in the same market meet a worker with probability  $qf(q)$ . Following the standard search literature, I assume that  $f(q)$  is twice differentiable, strictly decreasing and convex, with  $f(0) = 1$  and  $f(\infty) = 0$ . I also assume that  $qf(q)$  is strictly increasing and concave, approaching 1 as  $q$  converges to  $\infty$ . In addition, I assume that the elasticity of the matching function with respect to job creation  $\eta(q)$ , defined as  $\frac{qf'(q)}{f(q)}$  is concave, with  $\eta(\infty) = 1$ . This assumption is not necessary but simplifies the existence proof of an equilibrium.

## 2.2 Worker's Problem

I assume  $(r + \delta)k_2 < y_2 - y_1$  for all time in this paper. This assumption guarantees enough profits for employers to poach employed worker and ensures that job-to-job transition is possible. Without on-the-job search, hiring a high school or a college worker is identical for a non-college job employer given that they are equally productivity on these jobs. Therefore, labour market equilibrium separates high school workers and college workers efficiently given their different reservation value of unemployment.

Let  $s = \{\ell, i\} \in S$  denote the state of a worker of type  $i$  in labour force status  $\ell$  and  $S$  the set of all possible states of a worker. Let  $V(s)$  denote the discounted lifetime return of a worker in state  $s$ . Since a contract specifies the labour market status of its applicants, an employed and an unemployed worker search for different contracts. An unemployed worker receives benefit  $b$  and decides where to search after the search cost is realized. When an unemployed worker  $i$  searches in market  $x = \{\omega, j, (b, 0)\}$ , the probability of meeting an employer is  $f(Q(x))$  and the state of the worker becomes  $s_o = \{(\omega, j), i\}$  once matched. If  $s = \{(b, 0), i\}$ , then the value function  $V(s)$  satisfies:

$$V(s) = b + \mathbb{E}_c \max_{(\omega, j)} \left\{ (1-i)(1-j)c + f(Q(x)) \max \left\{ \frac{V(s)}{1+r}, \frac{V(s_o)}{1+r} \right\} + [1 - f(Q(x))] \frac{V(s)}{1+r} \right\} \quad (1)$$

Similarly, when a type  $i$  worker, currently employed as  $\ell = (\omega', j')$  searches for contract  $x = \{(\omega, j), \ell\}$ , the probability of meeting a poaching employer is  $f(Q(x))$ . Once the outside offer  $\omega$  is made, the incumbent employer can respond by making a counter-offer  $\omega_c$ . If the worker accepts the outside offer, she switches to the new employer  $j$  and forms a new match starting the next period with state  $s_o = \{(\omega, j), i\}$ . Otherwise, the worker remains matched with her current employer. Her state becomes  $s_c = \{(\omega_c, j'), i\}$  with new wage  $\omega_c$  if the counter-offer is accepted, otherwise the worker remains in state  $s = \{\ell, i\}$ . For this employed worker, the value function  $V(s)$  for  $s \neq \{(b, 0), i\}$  satisfies:

$$V(s) = \omega' + \frac{\delta V(s_u)}{1+r} + (1-\delta) \left\{ \max_{(\omega, j)} \left\{ f(Q(x)) \max \left\{ \frac{V(s)}{1+r}, \frac{V(s_o)}{1+r}, \frac{V(s_c)}{1+r} \right\} + [1 - f(Q(x))] \frac{V(s)}{1+r} \right\} \right\} \quad (2)$$

Denote by  $g(s, c)$  the search policy of a worker in state  $s$  with cost realization  $c$  ( $c = 0$  if the worker is employed), also denote by  $g^a(s, s_o, s_c)$  the acceptance policy such that  $g^a(s, s_o, s_c)$  is the probability that a worker in state  $s$  accepts the outside offer. When no counter-offer is received,  $\omega_c$  equals to 0.

### 2.3 Employer's Problem

Consider an employer matched with a worker of type  $i$  with contract  $x = \{(\omega, j), \ell\}$ . Let  $H(s)$  be the discounted sum of profits of the employer matched with the worker, whose state becomes  $s = \{(\omega, j), i\}$  once matched. If the match is not hit by a separation shock, the employer anticipates that the worker can search on-the-job. Taking as given the worker's optimal search policy  $g$ , the firm expects that while matched, the worker will receive an outside offer  $x' = g(s, 0)$  with probability  $f(Q(x'))$ . Once an outside offer is received, the employer can make a counter-offer  $\omega_c$ , taking as given the the acceptance rule of the worker  $g^a$ . If the worker accepts the outside offer, her state becomes  $s^o = \{x', i\}$  and the match is destroyed with zero value left for the employer. Otherwise, the worker stays with the incumbent employer. The state of the worker remains at  $s$  or changes to  $s_c = \{(\omega_c, j), i\}$ , if she accepts the counter offer. The value of the match for the employer  $H(s)$

satisfies the following:

$$H(s) = y_j - \omega + (1 - \delta) \left\{ f(Q(x')) \max_{\omega_c} \left\{ [1 - g^a(s, s_o, s_c)] \frac{H(s_c)}{1+r} \right\} + [1 - f(Q(x'))] \frac{H(s)}{1+r} \right\} \quad (3)$$

Denote the optimal retention policy of an employer by  $g^r(s, s_o)$ . Employers choose how many vacancies to create and where to locate them. Because contracts are conditional on workers' labour force status but not on their types, employers need to form expectations about the distribution of the types of applicants attracted to each posted contract. Let  $\mu(\cdot|x)$  be the conditional distribution function of the types of workers who are attracted by contract  $x$ . Let  $J(x)$  be the ex-ante return of posting contract  $x$  to an employer.  $J(x)$  is given by

$$J(x) = -k_j + Q(x)f(Q(x)) \sum_{i=1}^2 \mu(i|x) g^a(s^i, s_o^i, s_c^i) \frac{H(s_o^i)}{1+r} \quad (4)$$

where  $j$  is the job type posted. The states  $s^i, s_o^i, s_c^i$  of a worker of type  $i$  satisfy  $s^i = \{\ell, i\}$ ,  $s_o^i = \{\{\omega, j\}, i\}$  and  $s_c^i = \{g^r(s^i, s_o^i), i\}$  respectively for  $i \in \{1, 2\}$ .

## 2.4 Equilibrium Refinement

I now introduce a refinement that follows closely the equilibrium concepts in Chang (2012) and Guerrieri, Shimer, and Wright (2010). Without any constraints on off-equilibrium beliefs, the model has numerous equilibria. The idea of the refinement is that a deviating contract must attract workers who have the strongest incentive to apply. Formally, denote  $\tilde{V}(x, q, s)$  the expected return to a worker in state  $s = \{\ell, i\}$  searching in market  $x$  with queue length  $q$ . The incentive of a worker  $i$  applying to deviating contract  $x$  is determined by the longest queue that a worker is willing to accept:

$$q(x, i) \equiv \sup\{\tilde{q} \geq 0 : \tilde{V}(x, \tilde{q}, s) \geq V(s)\} \quad (5)$$

Note that  $\tilde{q} = 0$  if  $\tilde{V}(\hat{x}, \tilde{q}, s) \geq V(s)$  has no solution. For contract  $x$ ,  $q(x, i)$  is the longest queue that a worker  $i$  is willing to accept in order to deviate. If  $q(x, i)$  is positive, then the workers is indifferent between her equilibrium allocation and applying to contract  $x$  with queue length equal to  $q(x, i)$ . If  $x$  could attract some positive measure of workers, the type of the worker who has the

strongest incentive to apply—the type with the longest  $q(x, i)$ —must have been attracted. Then the belief function  $\mu(\cdot|x)$  must satisfy that only workers who have the strongest incentive to apply are attracted. That is, belief must satisfy

$$\mu(i|x) = 0 \text{ if } i \notin \arg \sup_{i \in I} \{q(x, i)\} \text{ for any } x \in X. \quad (6)$$

This refinement arises naturally as a single employer does not serve the entire market under bilateral matching. It implies that when more than one type of worker is attracted by an off-equilibrium contract, their longest acceptable queue must be identical. It also implies that the queue mapping must satisfy:

$$Q(x) \equiv \sup_{i \in I} q(x, i)$$

The queue length associated with off-equilibrium contracts is determined by the longest acceptable queue of the attracted worker type. To understand this, consider the following adjustment process: if an employer posts an off-path contract  $x$ , some workers might find it profitable to apply if the market queue is sufficiently small. As workers flow in, the worker-over-employer ratio, i.e. the queue length increases. The flow-in of type  $i$  workers stops when they are indifferent between their equilibrium market and market  $x$ . They prefer not to apply to the market if the worker-over-employer ratio continues to increase if other types of workers with stronger incentives still flow in. This process stops when the queue length of the market reaches the longest acceptable queue of the worker type with the strongest incentive to apply. Therefore, the longest accepted queue of workers attracted to the contract determines  $Q(x)$  for off-equilibrium contracts.

## 2.5 Equilibrium

**Definition 1** *A stationary competitive search equilibrium consists of a set of posted contracts  $X^* \subseteq X$ , a market queue mapping  $Q$ , a value function for workers  $V$ , a value function for employers  $H$ , a search policy  $g$ , an acceptance rule  $g^a$ , a retention policy  $g^r$ , a conditional distribution function  $\mu \cdot$ , a distribution  $\phi$  such that:*

- (i) *Workers optimize:  $V(s)$  satisfies (1) when  $s \in S^u$  and satisfies (2) for  $s \in S \setminus S^u$  with  $g(s, c)$*

is the associated policy function and  $g(s, c) \in X^*$  for all  $c \in \mathbb{R}$ ,  $s \in S$ ; and  $g^a(s, s_o, s_c)$  is the acceptance rule such that

$$g^a(s, s_o, s_c) = \arg \max_{a \in [0,1]} \{aV(s_o) + (1-a) \max\{V(s), V(s_c)\}\} \quad (7)$$

(ii) Employers optimize:  $H(s)$  satisfies (3) for all  $s \in S^e$  with  $g^r(s, s_o)$  the corresponding policy function;

(iii) For any  $x \in X^*$ , firms make zero profits:  $J(x) = 0$

(iv) Beliefs are consistent: for any  $x \in X^*$  and any state  $\{\ell, i\} \in S$ ,

$$\mu(i|x) = \frac{\phi(\{\ell, i\})}{\sum_{i \in \{1,2\}} \phi(\{\ell, i\})}$$

(v) Steady-State conditions are satisfied.

(vi) Beliefs satisfy equilibrium refinement:  $\mu(i|x)$  satisfies (6)

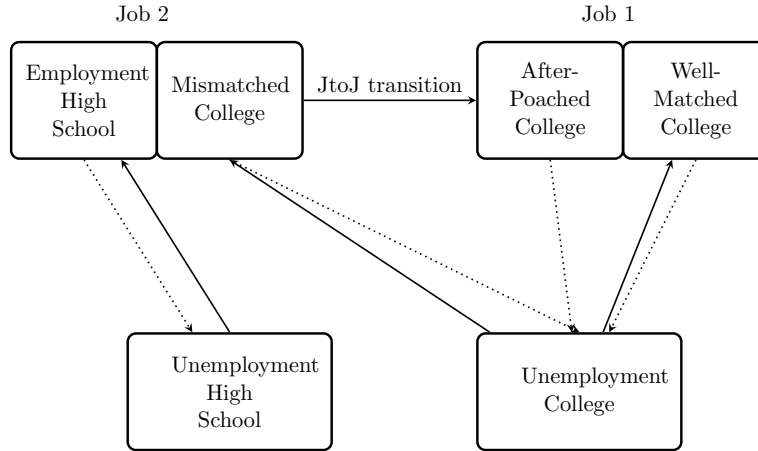
The general statement of the steady-state conditions is cumbersome and therefore not listed here, I provide the steady-state condition in the context of the equilibrium characterized below.

Condition (i) ensures that workers' search and acceptance policies are optimal for all states, taking as given the market queue length of all contracts and employer's optimal counter-offer strategy. Condition (ii) and (iii) ensures that, given the optimal search strategy of the worker, retention policies are optimal, and equilibrium contracts generates zero profits for employers. Condition (iii) ensures that for equilibrium contracts, employers beliefs are consistent with the workers search strategies through Bayes rule. Condition (v) ensures the law of motion for the aggregate state of the economy is stationary. Finally, condition (vi) restricts the off-equilibrium beliefs such that for any contract, only workers with the strongest incentives are attracted.

### 3 Equilibrium with Displacement

In this section, I study an equilibrium where the labour outcomes of high school workers are distorted by an inefficiently high unemployment rate. The equilibrium can be understood by studying

**Figure 1:** Labour Market Flow In Equilibrium



a set of constrained optimization problems.<sup>7</sup> The constrained optimization problems have an intuitive structure where workers maximize expected return to search subject to the condition that firms making non-negative profits and the incentive compatibility constraints of all types holding. After stating the problems, I show that a separating equilibrium can be supported with the solution of the problems and the equilibrium always exists. Under mild conditions, the incentive compatibility constraint of a college worker is binding and the equilibrium suffers from an adverse selection problem so that the unemployment rate of high school workers is inefficiently high.

Figure 1 presents the equilibrium flow of workers. High school workers search for non-college jobs only. Once employed, they do not search on-the-job. College workers can search for both college and non-college jobs when unemployed.<sup>8</sup> An employed college worker search on-the-job only if matched with a non-college job employer. Job-to-job transitions occur when a college worker switches from a non-college job employer to a college job employer. All employed workers face an exogenous separation shock. Once hit by the shock, a worker becomes unemployed.

In equilibrium, job-to-job transitions occur only when switching employers generates a produc-

<sup>7</sup>See a full discussion in Guerrieri, Shimer, and Wright (2010).

<sup>8</sup>See Shi (2002) for a competitive search equilibrium with heterogeneous skills, where high-tech firm receives both skilled and unskilled applicants, while a low-tech firm receives only unskilled applicants.

tivity gain.<sup>9</sup> This is because the search cost is a sunk cost for an employer who is already matched with a worker and it is optimal for the incumbent employer to counter offer up to the productivity of the match, once an outside offer is received by the employee. When there exists a negligibly small cost of making offers, an employer finds it not profitable to poach a worker who is equally productive with the current employer and with the poaching employer. Similarity, an incumbent employer does not make counter-offers if the worker is more productive with the poaching employer than in the current match. As a result, only college workers who are matched with non-college job employers search on-the-job. The search problems that need to be considered in the equilibrium are: the on-the-job search problem of an employed college worker, the search problem of an unemployed college worker for both college and non-college jobs and the search problem of an unemployed high school worker.

Consider first the on-the-job search problem of an employed college worker. Denote the value of a college worker who is currently employed in a non-college job with wage  $\omega$  by  $V(\{(\omega, 1), 2\})$ . The worker searches for a college job and faces the following search problem. The worker chooses across all pairs of  $(\omega', q)$  to maximize the expected return to search, subject to the constraint that the poaching employer makes non-negative profits and the poaching wage is no less than her current matching productivity. For any given pair  $(\omega', q)$ , the worker meets an employer with probability  $f(q)$  and forms a new match with value  $V(\{(\omega', 2), 2\})$  to the worker with the new employer. With probability  $1 - f(q)$ , the worker stays with the incumbent employer and her state remains unchanged.  $V(\{(\omega, 1), 2\})$  satisfies:

$$V(\{(\omega, 1), 2\}) = \omega + \max_{\omega', q} \left\{ f(q) \frac{V(\{(\omega', 2), 2\})}{1+r} + [1 - f(q)] \frac{V(\{(\omega, 1), 2\})}{1+r} \right\} \quad (\mathbf{P} - 1)$$

Subject to

$$-k_2 + qf(q) \left( \frac{y_2 - \omega'}{r + \delta} \right) \geq 0$$

and

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<sup>9</sup>See Delacroix and Shi (2006) for an analysis of wage ladders.



$$\omega' \geq y_1$$

Denote the problem by **(P – 1)**. The value of a match with a college job for a college worker satisfies

$$\frac{V(\{(\omega, 2), 2\})}{1+r} = \frac{\omega}{r+\delta} + \frac{\delta}{r+\delta} \frac{V(\{(b, 0), 2\})}{(1+r)}$$

which is equal to the discounted sum of wages plus the value of becoming unemployed ( $V(\{(b, 0), 2\})$ ) with probability  $\delta$ .

The first constraint is the non-negative profit constraint for a college job employer searching for a worker in the market with wage offer  $\omega'$  and queue length  $q$ . The employer pays cost  $k_2$  to create the job and with probability  $qf(q)$  she meets a worker. The expected profit of a match for the employer equals to  $\frac{y_2 - \omega'}{r + \delta}$ , which is the flow profit  $y_2 - \omega'$  of the match discounted by the common discount rate  $r$  and the job separation risk  $\delta$ . The second constraint restricts the equilibrium poaching wage to be no less than the worker's current productivity. Because the incumbent employer is able to make counter offers up to  $y_1$ , no poaching employer in equilibrium posts wages lower than  $y_1$  to attract a mismatched college worker.

Denote the solution to the above problem by  $\{q^e(\omega), \omega^e(\omega)\}$ . Notice that the solution to the on-the-job problem depends on the worker's current wage. The probability that a college worker switches to a college job employer depends on the wage  $\omega$  that she receives from the non-college job employer.

Next, I examine the problem of an unemployed college worker searching for a non-college job employer, denoted by **(P – 2)**. Denote the return to search for a non-college job to an unemployed college worker by  $V_2^1$ , such that

$$V_2^1 = \max_{\omega, q} \left\{ f(q) \frac{V(\{(\omega, 1), 2\})}{1+r} + [1 - f(q)] \frac{V(\{(b, 0), 2\})}{1+r} \right\} \quad (\mathbf{P} - 2)$$

s.t.

$$-k_1 + qf(q) \frac{y_1 - \omega}{r + \delta + (1 - \delta)f(q^e(\omega))} \geq 0$$

An unemployed college worker searches across all pairs of  $(\omega, q)$  subject to the constraint that the non-college job employer makes non-negative profits. With probability  $f(q)$ , the worker meets an employer and is matched with a non-college job and starts search on-the-job in the next period. With probability  $1 - f(q)$ , the worker does not find an employer and remains unemployed with value  $V(\{(b, 0), 2\})$ . Notice that the employer takes into consideration that the quit rate of the worker depends on the wage  $\omega$  and discounts the expected profit of a match with  $r + \delta + (1 - \delta)f(q^e(\omega))$ , where  $f(q^e(\omega))$  is the endogenous separation rate for the match.

Now consider the search problem of an unemployed college worker looking for a college job, denoted by **(P – 3)**. As in the problem **(P – 2)**, an unemployed college worker searches across all pairs of  $(\omega, q)$  subject to the constraint that the college job employer makes non-negative profits. Different from searching for a non-college job, a college worker does not search on-the-job once matched with a college job. Denote the return to search for a college job to an unemployed college worker by  $V_2^2$ , such that

$$V_2^2 = \max_{(\omega, q)} \left\{ f(q) \frac{V(\{(\omega, 2), 2\})}{1 + r} + [1 - f(q)] \frac{V(\{(b, 0), 2\})}{1 + r} \right\} \quad \textbf{(P – 3)}$$

s.t.

$$-k_2 + qf(q) \frac{y_2 - \omega}{r + \delta} \geq 0$$

The search problem is similar to the on-the-job search problem **(P – 1)**, except that the worker remains unemployed with value  $V(\{(b, 0), 2\})$  if she does not meet an employer within a period.

The reminding search problems of a college worker is to determine whether to search for a college job or a non-college job when unemployed. It is easy to see that there exists a cut-off level cost  $\bar{c}$ , such that a worker searches for a non-college job if and only if the realization of the search

cost is smaller than  $\bar{c}$ . Since the search cost  $c$  follows distribution with cumulative density function  $F(c)$ , an unemployed college worker searches en-ante for a college job with probability  $F(\bar{c})$  with return to search  $V_2^2$  and pays the cost. With probability  $1 - F(\bar{c})$ , the worker searches optimally for a non-college job with return to search  $V_2^1$  and does not pay the search cost. The value of unemployment to a college worker  $V(\{(b, 0), 2\})$  equals

$$V(\{(b, 0), 2\}) = b + [1 - F(\bar{c})]V_2^1 + F(\bar{c})[V_2^2 - \mathbb{E}(c|c < \bar{c})].$$

The cut-off cost  $\bar{c}$  must satisfy the condition that an unemployed college worker with cost realization  $\bar{c}$  is indifferent between searching for a college job and pay cost  $\bar{c}$  or searching for a non-college job without paying the cost. The cut-off  $\bar{c}$  is defined by the following equation:

$$V_2^2 - \bar{c} = V_2^1.$$

Now consider the search problem of an unemployed high school worker. Like college workers, an unemployed high school worker maximizes the expected return to search by choosing across all pairs  $(\omega, q)$  subject to the constraint that the employer makes non-negative profits. Unlike college workers, the optimization problem of an unemployed high school workers is also subject to an incentive compatibility (IC) constraint. For any given pair  $(\omega, q)$ , the worker meets an employer with probability  $f(q)$  and forms a match with value  $V(\{(\omega, 1), 1\})$ . Otherwise, the worker remain unemployed with value denoted by  $V(\{(b, 0), 1\})$ . Denote the search problem of an unemployed high school worker by **(P - 4)**, such that

$$V(\{(b, 0), 1\}) = b + \max_{\omega, q} \left\{ f(q) \frac{V(\{(\omega, 1), 1\})}{1+r} + [1 - f(q)] \frac{V(\{(b, 0), 1\})}{1+r} \right\} \quad (\mathbf{P} - 4)$$

s.t.

$$-k_1 + qf(q) \frac{y_1 - \omega}{r + \delta} \geq 0 \quad (\text{Non-negative profits constraint})$$

and

$$f(q) \frac{V(\{(\omega, 1), 2\})}{1+r} + [1 - f(q)] \frac{V(\{(b, 0), 2\})}{1+r} \leq V_2^1. \quad (\text{IC constraint})$$

The value of a match for a high school worker satisfies:

$$\frac{V(\{(\omega, 1), 1\})}{1+r} = \frac{\omega}{r+\delta} + \frac{\delta}{r+\delta} \frac{V(\{(b, 0), 1\})}{(1+r)}.$$

The IC constraint reflects the adverse selection problem and is the key to the displacement mechanism. Because of the non-discrimination against education restriction, contracts offered to high school workers can not exclude college workers from applying. At any given  $(\omega, q)$ , an employer expects less profits from matching with a college worker since the worker uses the job as a stepping-stone. The pair  $(\omega, q)$  that makes an employer earn zero profits by hiring a high school worker would generate negative profits if hiring a college worker. In equilibrium, no employer posts contracts to high school workers that would also attract college workers. The equilibrium contracts offered to high school workers must satisfy the condition that a college worker weakly prefers search in her own market with value  $V_2^1$  than searching for the same  $(\omega, q)$  as high school workers, the value of which equals the left-hand-side of the IC constraint. One can see from problem  $(\mathbf{P} - 1)$ , when the IC constraint is binding, the solution to an unemployed high school worker's search problem is no longer constrained optimal.

I now show that the properties of a competitive search equilibrium can be understood by studying the above optimization problems  $(\mathbf{P} - 1)$  to  $(\mathbf{P} - 4)$ . Proposition 1 below establishes the linkage between the problems and the equilibrium.

**Proposition 1** *i) Any equilibrium allocation must solve problems  $(\mathbf{P} - 1)$  to  $(\mathbf{P} - 4)$ . ii) A solution to problems  $(\mathbf{P} - 1)$  to  $(\mathbf{P} - 4)$  can be supported as an equilibrium.*

The proof of this proposition follows closely after Guerrieri, Shimer, and Wright (2010) and can be found in the appendix. Statement i) says that any solution to the set of problems generates an equilibrium. It is easy to see that the search problems in markets with college jobs are the standard competitive search problems without adverse selection. In markets with non-college jobs, however, the equilibrium belief function  $\mu(\cdot|x)$  is not degenerate because of the adverse selection problem. To show that workers maximize the expected return to search given the belief function and the corresponding queue mapping function, I consider a truncated belief function. For any wage post

that is lower than the wage of an “indifference allocation”, with which both the non-negative profit constraint and IC constraint in problem  $(\mathbf{P} - 4)$  are binding, the employers expect only college workers to apply; otherwise the employers expect only high school workers to apply. With this belief function and other corresponding equilibrium objects, the solution to problems  $(\mathbf{P} - 1)$  to  $(\mathbf{P} - 4)$  can be supported as a separating equilibrium, where college and high school workers search in different markets for a non-college employer.

The second statement of Proposition 1 says that one can find any equilibrium by solving problems  $(\mathbf{P} - 1)$  to  $(\mathbf{P} - 4)$ . Notice that problems  $(\mathbf{P} - 2)$  and  $(\mathbf{P} - 4)$  are constructed under the assumption that an employer with a non-college job expects to meet either a college worker or a high school worker but not a mixed distribution of them. Therefore, statement ii) implies that any equilibrium must be separating. The explanation of the no pooling equilibrium is as follows. If there were pooling in equilibrium, employers could attract high school workers by offering a higher wage, thereby breaking the proposed equilibrium. This is because unlike college workers who search for non-college jobs because they are a stepping stone to better jobs, high school workers are more willing to remain unemployed and search for jobs offering a relatively higher wage. The rational refinement on the off-path belief function ensures that a deviating wage attracts the type of workers who have the strongest incentives to apply. Therefore, offering a higher wage attracts the desired type of workers, which in this case are the high school workers, and is a profitable deviation.

Given that any solution to the set of problems  $(\mathbf{P} - 1)$  to  $(\mathbf{P} - 4)$  generates an equilibrium and any equilibrium must solve the problems, the existence of a separating equilibrium follows immediately.

**Proposition 2** *There is a separating equilibrium.*

The sorting mechanism of the separating equilibrium is as follows. The market sorts unemployed workers by education according to their tolerance to remaining unemployed. College workers are less willing to remain unemployed, since they search for non-college jobs because they are a stepping stone to better jobs, so they are willing to search for jobs offering a relatively lower wage as long as the job finding rate is sufficiently high. In a separating equilibrium, the market makes the jobs

that attract high school workers sufficiently hard to get so as to discourage educated workers from applying.

The next proposition states the main result of the paper. Under mild condition, the separating equilibrium suffers from an adverse selection problem and the labour market outcomes of high school workers are distorted.

**Proposition 3** *There is a number  $\bar{k} > 0$  such that for any  $k_2 \in (0, \bar{k})$ , an equilibrium suffers from adverse selection, and the unemployment rate of high school workers is inefficiently high.*

I show that when the cost of posting a college job is sufficiently small, the IC constraint of a college worker is binding. In this case, the constrained efficient allocation to a high school worker, which is the solution to the optimizing problem where high school workers maximize the expected return to search subject to employers making zero-profits, also attracts college workers. This means when the IC constraint is binding, no employer offers the wage that a high school worker finds optimal to apply. Instead, the market makes the jobs that attract high school workers sufficiently hard to get in order to exclude college workers from applying. The reason is that when searching for a non-college job, college workers have lower tolerance to remaining unemployed than high school workers. As a market solution to the adverse selection problem, the market sorts workers with different career paths by distorting the matching probabilities of high school workers.

In order for the market to distort the equilibrium matching rate and separate workers, the wage offered to high school workers is higher than its efficient level, even though the return to search, which is the discounted total labour income of a high school worker over time, is inefficiently low. In fact, the model implies that the skill premium, measured as the wage differential between high school and college workers, underestimates the true returns to education. This is because the true returns to education take into account not only the wage when a worker is employed, but also the job finding probability when a worker is unemployed, as well as the discounted value of future opportunities when a worker is employed. In this model, even though the skill premium is inefficiently low, as the wage of a high school workers is above its efficient level, the true returns to education are inefficiently high.

In the competitive framework with no search frictions, less-educated workers are replaced by more productive high-skilled workers in jobs that are less skill intensive and in routine tasks (see Acemoglu and Autor (2011) and Beaudry, Green, and Sand (2013)). Yet firm-level study has found no evidence supporting that firms upgrade their work force for low-skilled jobs in economic downturns (Gautier, Van den Berg, Van Ours, and Ridder, 2002). In these models, no externality is generated by high-skilled workers employed in less-skilled jobs since replacing less-educated workers by high skilled ones is an efficient market adjustment. In the search literature with random matching models, high-skilled workers create search externalities by applying to low-skilled jobs, which affects the creation of these jobs (see Gautier (2002), Albrecht and Vroman (2002) Dolado, Jansen, and Jimeno (2009)). These externalities rely crucially on the assumption that high-skilled workers and less-educated workers are matched randomly with employers offering low-skilled jobs. However, the observation that the proportions of college workers vary across non-college occupations (Vedder, 2012) suggests that college workers do not search randomly for low-skilled jobs. Without imposing the assumption of direct competition, it is unclear what is the mechanism of the displacement effect.

In this paper, the search behaviour of college workers generates a negative spillover effect on the unemployment rate of high school workers. The effect is similar to the crowding-out effect in the literature, but the mechanism is quite different. The high unemployment rate of high school workers is a market response to an adverse selection problem, where employment contracts cannot exclude college workers seeking non-college jobs as a stepping-stone. The displacement of high school workers is inefficient and is not caused by productivity-driven adjustments. Even if college workers are less productive in non-college jobs, the distortions for high school workers still exist. The mechanism does not rely on the direct competition assumption between the workers either. This means even without observing college and high school workers competing for the same jobs, the negative spillover effects caused by over-qualified college workers could still harm high school workers. In the next section, I proceed by discussing the implication of this model on the post-second vocational education and training.

## 4 Vocational Education

In the previous section, I showed that the labour market responses to the adverse selection problem distorts the labour market outcomes of high school workers. In this section, I discuss the implication of the model. I argue that post-secondary vocational education has value as an entry barrier to college workers and is an institutional solution to the adverse selection problem.

Vocational education here refers to nonbaccalaureate post-secondary level education that focuses primarily on providing occupationally specific preparation. In the United States, vocational education and training (VET) has been growing quickly in the past few decades. As shown in the introduction of the paper, the total number of VET certificates awarded per year has increased over 60 percent from 2000 to 2010, growing much faster than the 34 percent increase of bachelor degrees.<sup>10</sup> VET credentials take the form of either a post-secondary certificate or an associates degree and are provided mostly by community colleges in the U.S.

In this section, I first argue that the fast-growing population of VET certified workers can be understood as a response to the adverse selection problem introduced in the previous sections. By acting as an entry barrier to college workers, vocational education helps improve the labour market outcomes of high school workers and increases labour market efficiency. I formalize this intuition by extending the baseline model to include educational choices. I show that vocational education is a useful labour market institution that mitigates the adverse selection problem. I also show that in contrast to the competitive models where barriers to entry are viewed as rent-seeking distortions to the market, introducing costly vocational education as an entry barrier to the labour market is welfare improving. Finally, I perform a numerical simulation and examine the effects of skill-biased technological changes as well as an increase in college tuition on the changes in educational choices.

### 4.1 Vocational education as an Entry Barrier

With the insight of the mechanism presented in this paper, I argue that the displacement of high school workers caused by the adverse selection problem creates a demand for vocational education.

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<sup>10</sup>National Center for Education Statistics (NCES).



The intuition behind such a demand is that vocational education has value as an entry barrier to college workers. By requiring vocational education, a non-college job employer can exclude college workers from seeking stepping-stone opportunities and can therefore offer non-distorted wages to high school workers. By obtaining vocational education, high school workers gain access to markets that do not suffer from the adverse selection problem and improve their labour market performance. As long as the cost of vocational education does not exceed the gain from receiving non-distorted labour market allocations, high school workers have incentives to pursue vocational education.

The adverse selection problem discussed in this paper and the role of vocational education as an enter barrier are similar to the standard Spence's signalling model of education (Spence, 1973). In the signalling model, workers' unobserved productivity creates an adverse selection problem and educational credentials can be used to signal worker's abilities. In this paper, the on-the-job search and the incompleteness of contracting to exclude over-educated applicants are the cause of the adverse selection problem. Notice that in this environment, workers with more education are the less desired type to the employers, because they are more likely to quit a lower-skilled job in favour of high-skilled jobs. In this sense, on-the-job search changes the direction of the adverse selection problem in terms of educational levels. Using vocational education, employers can condition job offers on required training and exclude college workers from seeking stepping-stones. Therefore, vocational education helps mitigate the adverse selection, like the role of educational credentials in Spence's model, by resolving the contracting limitation of non-discrimination against education. Next, I formally present this intuition by extending the baseline model to include educational choices and vocational education.

## 4.2 A labour market with vocational education

Consider now a third type of jobs: VET jobs (denoted by  $v$ ). The difference between non-college jobs and VET jobs is that VET jobs require VET credentials while non-college jobs do not require any specific educational credential and can be performed by all workers. To separate the value of VET as an entry barrier from productivity, I assume in the model that VET jobs and non-college jobs are equally productive such that  $y_2 > y_1 = y_v$ .

The distribution of workers' educational attainment is determined endogenously. All workers are ex-ante identical. They make a one-time educational choice at period zero before entering the labour market. The choice is made based on the cost and the expected return of education. Let  $e$  denote the cost of post-secondary education to a worker, where  $e$  follows a distribution with cumulative function  $F_e(\cdot)$ , and is i.i.d. across all workers. A worker chooses whether to pursue post-secondary education and within the post-secondary educational system whether to pursue a VET credential or a bachelors degree. A worker with cost  $e$  needs to pay  $\alpha e$  for a bachelor degree and  $\beta e$  for VET with  $\alpha > \beta$ . In this model, all workers have high school diplomas.

The search problem of high school workers and college workers are identical to the baseline model. A worker with a VET credential can apply for both non-college jobs and VET jobs. However, the worker in equilibrium applies to VET jobs only and does not search on-the-job once matched.<sup>11</sup> Unlike high school workers, however, the search problem for workers with VET is not subject to an adverse selection problem. Therefore workers with VET credentials receive efficient allocations by maximizing the expected return to search subject to employers making zero-profits. Denote by  $U_h, U_v, U_c$  the equilibrium return to search for a high school worker, a worker with a VET credential and a worker with a bachelor degree respectively. Within an educational group, all workers are identical.

To determine the distribution of workers, consider the following two cutoffs. Let  $\bar{e}$  be the cutoff cost to post-secondary education so that a worker with educational cost  $\bar{e}$  is indifferent between pursuing VET or obtaining a high school diploma only. The cutoff to post-secondary education  $\bar{e}$  must satisfy

$$U_h = U_v - \beta \bar{e} \tag{8}$$

Similarly, let  $\underline{e}$  be the cutoff so that a worker with educational cost  $\underline{e}$  is indifferent between obtaining a VET credential or a bachelor degree. The cutoff  $\underline{e}$  must satisfy:

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<sup>11</sup>For a worker with a VET credential, the cost of VET is a sunk cost and the optimal return to searching for VET jobs weakly dominates the return to searching for non-college jobs. Once matched with a VET job, there is no option value from on-the-job search, therefore the worker does not search on-the-job.

$$U_v - \beta \underline{e} = U_c - \alpha \underline{e} \quad (9)$$

If  $\bar{e} \leq \underline{e}$ , a worker chooses either to attain post-secondary education at university level or obtain only a high school diploma with no demand for VET. In the following proposition, I show that when the labour market suffers from the adverse selection problem and the cost of VET is not too large, the demand of VET is positive.

**Proposition 4** *There is a level  $\bar{\beta} \in (0, \alpha)$  such that for any  $\beta \in (0, \bar{\beta})$ , there exists an equilibrium with positive vocational education, for any  $k_1 \in (0, \bar{k})$ , with  $\bar{k}$  defined in Proposition 3.*

When the cost of VET is not too large relative to the cost of university education,  $\bar{e} > \underline{e}$ . In this case, some workers find it optimal to pursue VET. In equilibrium, workers with an educational cost  $e$  smaller than  $\underline{e}$  go to university and can search for both non-college jobs and college jobs after graduation. Workers with a cost above  $\bar{e}$  do not pursue post-secondary education and can only apply to non college jobs. Workers with a educational cost in between  $\underline{e}$  and  $\bar{e}$  choose VET. Workers with VET credentials can apply to both non-college and VET jobs, but choose only VET jobs in equilibrium. In this model, while the productivity gains of university education drives some workers to pursue post-secondary education, the entry barrier role of VET also encourages workers with only a high school diploma to pursue post-secondary education. Next I discuss the welfare implications of VET.

At period 0, all workers are identical. Consider the situation where there is a demand for VET in the market. A worker makes educational choices based on the realization of her educational cost. With probability  $F_e(\underline{e})$ , the worker goes to universities for a college degree and pays the educational cost  $\alpha e$ . With probability  $1 - F_e(\bar{e})$ , the worker holds a high school diploma only. With probability  $F_e(\bar{e}) - F_e(\underline{e})$ , the worker attains post-secondary education for VET and pays the educational cost  $\beta e$ . The ex-ante utility of a worker denoted by  $U_0$  follows:

$$U_0 = F_e(\underline{e})U_c + [F_e(\bar{e}) - F_e(\underline{e})]U_v + [1 - F_e(\bar{e})]U_h - \beta \mathbb{E}[e | \underline{e} \leq e < \bar{e}] - \alpha \mathbb{E}[e | e \leq \underline{e}] \quad (10)$$

Since all employers in equilibrium make zero ex-ante profits, I can compare the total welfare

of different economies by comparing the levels of  $U_0$ . Consider two labour markets with different educational institutions: one does not have VET and the other one offers VET which is costly. The following proposition shows that VET improves welfare:

**Proposition 5** *The optional to undertake VET makes workers better off ex-ante, for  $k_1 \in (0, \bar{k})$  and  $\beta \in (0, \bar{\beta})$ , where  $\bar{k}$  and  $\bar{\beta}$  are given by Proposition 3 and Proposition 4, respectively.*

Introducing costly VET acting as an entry barrier for college workers can improve welfare by solving the adverse selection problem. VET attracts high school workers and workers who would choose university education when VET is absent. The gain from VET for high school workers is that workers with VET credentials gain access to jobs that exclude college workers and receive labour market outcomes that are not distorted by the adverse selection problem. Some workers who would pursue university education in the absence of VET also benefit from VET. These workers have relatively low educational costs amongst all workers so that they choose university level education. However, they have relatively high costs amongst college students so that their gain from the university education net the cost is small. With the option of VET, these workers benefit from applying to VET because even though the return to VET is less than the return to university credentials, the cost they save from pursuing VET instead of university credentials surpasses the difference in returns. So long as the cost to VET is not too large relative to gaining a bachelors degree, introducing VET to the labour market is strictly welfare improving.

One commonly debated form of entry barriers is occupational licensing. According to Kleiner and Krueger (2013), in 2008 nearly 30 percent of the workers with more than high school education, but not a bachelors degree, were required to hold a license. Many popular fields of VET such as health care and collective “trades” fields of manufacturing, construction, repair, and transportation, also prepare students for obtaining occupational licensing or certifications. One concern with occupational licensing is that as an entry barrier, occupational licensing may cause job losses by increasing employment costs (see Kleiner (2005)). This is because in a market with perfect competition, an entry barrier is a rent-seeking distortion to the market. However, using the insight of this paper, I argue that an entry barrier also has an important benefit to employment. In particular, imposing entry barriers on certain occupations provides protections for low to middle-skilled

workers from the competition of over-qualified college workers seeking stepping-stones.

Notice that the mechanism of the displacement of high school workers through adverse selection is the key to the welfare implication on vocational education. If displacement of high school workers happens in an efficient labour market (see Acemoglu and Autor (2011), Beaudry, Green, and Sand (2013) ), a barrier to entry creates monopoly power and is a distortion to the market. Introducing new tasks or jobs that are equally productive as existing ones but requiring additional training cost is not welfare improving. Even though low skilled workers could benefit from gaining access to jobs that exclude competition of college workers, the economy losses from incurring training cost without increasing productivity.

### 4.3 Post-Secondary Education and Skill-Biased Technological Changes

In this section, I use some simulations of the model to help understand the effects of important labour market trends on the increase of post-secondary education in the past thirty years. In particular, I consider the effects of skill-biased technological changes (SBTC), which complement skilled cognitive jobs but substitute less-skilled routine jobs,<sup>12</sup> on educational choices. I also discuss the impact of an increase in college tuition on VET.

#### 4.3.1 Benchmark Parameter Values

Assume the matching technology is given by  $M(u, v) = \frac{uv}{u+r}$ . Consider the following parameter values.<sup>13</sup> The productivity of non-college jobs  $y_1$  is normalized to 1 and the productivity of college jobs  $y_2$  equals 2.5. I relax the assumption that the productivity of VET jobs is equal to the productivity of non-college jobs and set  $y_v = 1.5$ . The flow income of unemployment  $b$  is set to be 0.1. Time is measured in quarters. Let  $r = 0.01$ , which is equivalent to an annual discount rate of 0.96. Given that job-to-job transition is endogenous in this model, the exogenous separation rate is set to be  $\delta = 0.05$ , consistent with the finding of Sahin, Song, and Hobijn (2010). I assume the search cost  $c$  follows an exponential distribution with parameter  $\lambda = 1$  and the educational cost

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<sup>12</sup>See Autor, Levy, and Murnane (2003) on routine jobs, Goos, Manning, and Salomons (2014) on skill biased technological change versus globalization and Autor, Dorn, Hanson, and Song (2014) on exposure to international trade.

<sup>13</sup>I found the IC constraint is always binding in all simulation examples considered.

$e$  follows beta distribution with parameter  $(4, 4)$ . The cost of posting a vacancy  $k$  is equal to 0.2. The educational cost rate  $\alpha$  and  $\beta$  are set to be 450 and 135 respectively, so that the educational attainment of the benchmark economy has 10 percent college graduates, 10 percent of VET workers and 80 percent high school workers, matching roughly the distribution of educational attainment in the 1970s.<sup>14</sup>

Column 2 in Table 2 presents the distribution of educational attainment and the key labour market outcomes of different workers in the benchmark economy. The economy has 80 percent high school workers and 10 percent each of college and VET workers. Workers with more education have a higher wage and lower unemployment rate. The college-versus-high-school wage ratio is 2.47, matching the college-versus-high-school earnings ratio in the early 80s (Acemoglu and Autor, 2011).

Next, I turn to simulations. I consider a positive shock to the productivity of college jobs, an negative shock to the productivity of non-college jobs and a scenario that combines both shocks with an increase in college tuition. The increase in productivity of VET jobs has one word effects and is not examined here.

**Table 2**

	Benchmark	Scenario 1	Scenario 2	Scenario 3
<b>Education Attainment</b>				
College	10%	98%	10%	33%
VET	10%	0	52%	30%
High School	80%	2%	38%	37%
<b>Unemployment Rate</b>				
College	6.0%	5.6%	6.0%	5.6%
VET	6.5%	6.5%	6.5%	6.5%
High School	9.6%	9.7%	10.3%	10.7%
<b>Mean Wage</b>				
College	2.42	4.92	2.42	4.92
VET	1.46	1.46	1.46	1.46
High School	0.98	0.98	0.68	0.68

<sup>14</sup>U.S. Bureau of Labour Statistics. I use here the group of workers with some college as well as workers with associate degrees to proxy VET workers.

### 4.3.2 Increased productivity of college jobs

Consider an increase in the productivity of college jobs with  $y_2 = 5$ . The results are presented in the second column of table 2 under Scenario 1. With a 100 percent increase in the productivity of college jobs while keeping the educational cost to university constant, 98 percent of workers would choose to become a college graduate and no worker prefers VET. The positive shock to college jobs has two effects on a worker's ex-ante educational choice. Increasing the productivity of college jobs raises the return to university education directly with a large increase in the wage and a mild decrease in the unemployment rate. It also lowers the return to search for a high school worker through the adverse selection effect. When the ex-ante return to search for an unemployed college worker is higher, the worker becomes more "picky" about jobs even when applying for a stepping-stone job. As a result, the worker is more patient to remain unemployed and is less willing to accept a job with a lower wage, making it more difficult for the market to separate workers. Further distortions on high school workers are reflected in the increase of the unemployment rate and a negligible wage rise. The second effect, however, is quantitatively dominated by the first one.

### 4.3.3 Decreased productivity of non-college jobs

Now consider a fall on the productivity of non-college jobs by 30 percent so that  $y_1 = 0.7$ . This fall can be thought of as automation and offshoring processes that substitute for routine-based less-skilled workers. The results are presented in the third column of table 2 under Scenario 2. The negative shock reduces the return to search to an unemployed high school worker by decreasing the wage and increasing the unemployment rate. It increases the incentive for a high school graduate to obtain post-secondary education. However, the increase in post-secondary education is captured entirely by a 42 percent increase in VET without changing the fraction of university graduates. The negative shock on non-college jobs has little effect on the return to search for an unemployed college worker. This is because when a non-college job is less productive, fewer unemployed college workers choose to apply for a non-college job.

#### 4.3.4 Increased university tuition

Finally, I consider a scenario with the two shocks discussed previously and a 100 percent increase in university cost  $\alpha$ . As shown in the last column of Table 2 under Scenario 3, the economy has 33 percent college workers, 30 percent VET workers and 37 percent high school workers, similar to the education attainment in the U.S. in 2010.<sup>15</sup> The relative wage ratios between college workers versus VET workers and high school workers are 3.4 and 7.2 respectively. While the increased productivity in college jobs attracts workers to enrol in universities, the negative shock to non-college jobs and the increase in the cost to colleges relative to the cost to VET encourage workers to attain VET instead.

I relate these results to the argument of Autor et al. (2010) who points out that one puzzle in the U.S. labour market is that given the steep rise in the college-versus-high-school earnings ratio, the relative supply of college-educated workers is not growing fast enough. This paper shows that much of the increase in post-secondary education has taken the form of non-bachelor level post-secondary VET. This is because skill-biased technological changes have increased not only the return to college relative to high school but also the return to non-university VET. Together with an increase in college tuition and costs, VET becomes another popular educational investment amongst students.

## 5 Conclusion

In this paper, I argue that the labour market generates inefficient unemployment of high school graduates as a mechanism to separate high school graduates from overqualified college graduates searching for some types of routine jobs. This is how the labour market resolves the adverse selection problem arising from the fact that employment contracts in those routine jobs do not discriminate between high school and college graduates. In this context, the demand for vocational education arises because it allows employers to exclude applicants who treat the job as a stepping-stone. As a result, a vocational credential provides high school workers with access to markets that do not suffer from the distortion of over-educated workers and improve their labour market outcomes.

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<sup>15</sup>U.S. Bureau of labour Statistics.



Given the steep rise in the college-versus-high-school earnings ratio, one puzzle in the U.S. labour market is that the relative supply of college-educated workers is not growing fast enough (Autor et al., 2010). In fact, the biggest growth in education in the past decades was the post-secondary vocational education. This paper addresses this puzzle by arguing that vocational education has a higher market value than it is commonly thought. In this content, skill-biased technological change can explain the rise of post-secondary vocational education over the past decades.

## Appendix

### Proof of Proposition 1

I first show that a solution exists to problem  $(\mathbf{P} - 1)$  to  $(\mathbf{P} - 4)$ .

**Lemma 1** *There exists a solution to  $(\mathbf{P} - 1)$  to  $(\mathbf{P} - 4)$ .*

To show Lemma 1, consider first the problem  $(\mathbf{P} - 3)$  of college workers. Denote  $V(\{(b, 0), 2\}) \equiv V_2$ . The first order conditions with respect to  $q$  and  $\omega$  jointly gives

$$\frac{V(\{(\omega, 2), 2\})}{1+r} - \frac{V_2}{1+r} = \frac{1-\eta(q)}{\eta(q)} \frac{k_2}{qf(q)} \quad (11)$$

Substituting  $\frac{V(\{(\omega, 2), 2\})}{1+r}$  and the zero-profit condition and denote the solution to this market  $(\omega_2^2, q_2^2)$  which solves the following equations for any given  $V_2$

$$\frac{y_2}{r+\delta} - \frac{r}{r+\delta} \frac{V_2}{1+r} = \frac{k_2}{\eta(q_2^2)q_2^2 f(q_2^2)} \quad (12)$$

$$\omega_2^2 = y_2 - \frac{(r+\delta)k_2}{q_2^2 f(q_2^2)} \quad (13)$$

The expected return to search for a college workers  $V_2^2$  can be expressed as

$$V_2^2 = \frac{1}{r} \left[ y_2 - \frac{k_2(r+\delta)}{\eta(q_2^2)q_2^2 f(q_2^2)} \right] + \frac{1-\eta(q_2^2)}{\eta(q_2^2)} \frac{k_2}{q_2^2} \quad (14)$$

Next consider the on-the-job search problem  $(\mathbf{P} - 1)$ . Ignore the last constraint, the first order condition for  $\omega'$  and  $q$  jointly gives

$$\frac{\omega'}{r+\delta} + \frac{\delta}{r+\delta} \frac{V_2}{1+r} - \frac{V(\{(\omega, 1), 2\})}{1+r} = \left[ \frac{1-\eta(q)}{\eta(q)} \right] \left( \frac{y_2 - \omega'}{r+\delta} \right) \quad (15)$$

Substitute the stationary  $V(\{(\omega, 1), 2\})$  from (28), an interior solution  $q^e(\omega)$  and  $\omega^e(\omega)$  satisfies

$$\frac{\omega^e(\omega) - \omega}{r+\delta + (1-\delta)f(q^e(\omega))} = \left[ \frac{1-\eta(q)}{\eta(q)} \right] \left( \frac{y_2 - \omega'}{r+\delta} \right) \quad (16)$$

together with the zero profit condition

$$\omega^e(\omega) = y_2 - \frac{k_2(r+\delta)}{q^e(\omega)f(q^e(\omega))} \quad (17)$$

Denote  $q_a$  such that

$$q_a f(q_a) \left( \frac{y_2 - y_1}{r+\delta} \right) = k_2 \quad (18)$$

$q^e(\omega) \geq q_a$  if and only if  $\omega^e(\omega) \geq y_1$ . Also let  $q_b$  be such that

$$\frac{y_2 - y_1}{r + \delta} = \frac{k_2}{q_b f(q_b)} \left[ 1 + \frac{1 - \eta(q_b)}{\eta(q_b)} \frac{r + \delta + (1 - \delta)f(q_b)}{r + \delta} \right] \quad (19)$$

then  $q^e(\omega) \leq q_b$  if and only if  $\omega \leq y_1$ . For any  $\omega \in [0, y_1]$ , the pair  $(\omega^e(\omega), q^e(\omega))$  is uniquely given by (17) and (20) with  $y_1 \leq \omega^e(\omega) < y_2$  and  $0 \leq q_a \leq q \leq q_b < \infty$ , where (20) is given by

$$\frac{\omega^e(\omega) - \omega}{r + \delta + (1 - \delta)f(q^e(\omega))} \geq \left[ \frac{1 - \eta(q)}{\eta(q)} \right] \left( \frac{y_2 - \omega'}{r + \delta} \right) \quad (20)$$

with equality if  $\omega^e(\omega) > y_1$ . Rewrite equations (16) so that the wage from the off-job search problem is a function of the quit rate

$$\omega(q^e) = y_2 - \frac{k_2}{q^e f(q^e)} \left( r + \delta + \frac{1 - \eta(q^e)}{\eta(q^e)} [r + \delta + (1 - \delta)f(q^e)] \right) \quad (21)$$

Substituting  $\omega$  and transform the surplus to the worker as a function of the quit rate. Denote  $\hat{V}(q^e) \equiv \frac{V(\{(\omega, 1), 2\})}{1 + r}$

$$\hat{V}(q^e) = \frac{y_2}{r + \delta} - \frac{k_2}{q^e f(q^e) \eta(q^e)} + \frac{\delta}{r + \delta} \frac{V_2}{1 + r} \quad (22)$$

$$\frac{d\hat{V}(q^e)}{dq^e} = \frac{k_2}{q^e f(q^e)} \left( \frac{1}{q^e} \frac{1 - \eta(q^e)}{\eta(q^e)} + \frac{\eta'(q^e)}{\eta^2(q^e)} \right) \quad (23)$$

Next consider the following transformed problem from **(P - 2)**. Using (22) the off-job search problem for a college worker looking for a non-college job can be reformulated as

$$\max_{q', q} \left\{ f(q) \hat{V}(q') + [1 - f(q)] \frac{V_2}{1 + r} \right\} \quad (\mathbf{P} - 2')$$

s.t.

$$-k_1 + qf(q) \left[ \hat{S}(q') - \hat{V}(q') - \frac{V_2}{1 + r} \right] \leq 0$$

Where  $\hat{S}(q')$  is the total surplus of the match and  $\hat{S}(q') - \hat{V}(q') - \frac{V_2}{1 + r} = \frac{y_1 - \omega(q')}{r + \delta + (1 - \delta)f(q')}$ , with

$$\begin{aligned} \frac{d \left( \hat{S}(q^e) - d\hat{V}(q^e) + \frac{V_2}{1+r} \right)}{dq^e} &= \frac{(1 - \delta)f'(q^e)}{[r + \delta + (1 - \delta)f(q^e)]^2} \left( y_2 - y_1 - \frac{k_2(r + \delta)}{q^e f(q^e)} \right) \\ &\quad - \frac{k_2}{q^e f(q^e)} \left[ \frac{1 - \eta(q^e)}{q^e} \left( \frac{1 - \eta(q^e)}{\eta(q^e)} + \frac{r + \delta}{r + \delta + (1 - \delta)f(q^e)} \right) + \frac{\eta'(q^e)}{[\eta(q^e)]^2} \right] \end{aligned} \quad (24)$$

I now solve the problem **(P - 2')** with the first order conditions with respect to  $q'$  and  $q$  given by

$$\lambda q = -\frac{\frac{d\hat{V}(q')}{dq'}}{\frac{d}{dq'}\left(\hat{S}(q') - \hat{V}(q') - \frac{V_2}{1+r}\right)} \quad (25)$$

$$\hat{V}(q') - \frac{V_2}{1+r} = \lambda q \left[ \frac{1 - \eta(q)}{\eta(q)} \right] \frac{k_1}{qf(q)} \quad (26)$$

Denote the solution to problem  $\mathbf{P} - \mathbf{2}'$  as  $(q^e, q_2^1)$ , which satisfies

$$\frac{y_2}{r + \delta} - \frac{k_2}{\eta(q^e)q^e f(q^e)} - \frac{r}{r + \delta} \frac{V_2}{1+r} = \lambda q_2^1 \left[ \frac{1 - \eta(q_2^1)}{\eta(q_2^1)} \right] \frac{k_1}{q_2^1 f(q_2^1)} \quad (27)$$

The return to search for a non-college job  $V_2^1$  for college worker satisfies:

$$V_2^1 = \frac{1}{r} \left( y_2 - \frac{k_2(r + \delta)}{\eta(q^e)q^e f(q^e)} - \lambda q_2^1 \left[ \frac{1 - \eta(q_2^1)}{\eta(q_2^1)} \right] \frac{k_1(r + \delta)}{q_2^1 f(q_2^1)} \right) + \lambda q_2^1 \left[ \frac{1 - \eta(q_2^1)}{\eta(q_2^1)} \right] \frac{k_1}{q_2^1} \quad (28)$$

Substituting  $V_2^1$  and  $V_2^2$  from (28) and (14) into equation (3), the cut-off  $\bar{c}$  solves

$$\bar{c} = \left[ \frac{1 - \eta(q_2^2)}{\eta(q_2^2)} \right] \frac{k_2}{q_2^2} - \lambda q_2^1 \left[ \frac{1 - \eta(q_2^1)}{\eta(q_2^1)} \right] \frac{k_1}{q_2^1} \quad (29)$$

From equation (3), the expected return to search for a college worker  $V_2$  follows

$$V_2 - b = V_2^2 - \bar{c} + \nu(\bar{c}) = V_2^1 + \nu(\bar{c}) \quad (30)$$

where  $\nu(\bar{c}) = F(\bar{c})[\bar{c} - \mathbb{E}(c|c \leq \bar{c})]$ . The three equations listed below together equation (29) solves the search problem of a college worker with unknowns  $\bar{c}$ ,  $q_2^1$ ,  $q_2^2$  and  $q^e$ .

$$\frac{y_2 - b + \bar{c} - \nu(\bar{c})}{r + \delta} = \frac{k_2}{\eta(q_2^2)q_2^2 f(q_2^2)} + \left[ \frac{1 - \eta(q_2^2)}{\eta(q_2^2)} \right] \frac{k_2}{(r + \delta)q_2^2} \quad (31)$$

$$\frac{y_2 - b - \nu(\bar{c})}{r + \delta} - \frac{k_2}{\eta(q^e)q^e f(q^e)} = \lambda q_2^1 \left[ \frac{1 - \eta(q_2^1)}{\eta(q_2^1)} \right] \frac{k_1}{f(q_2^1)q_2^1} + \lambda q_2^1 \left[ \frac{1 - \eta(q_2^1)}{\eta(q_2^1)} \right] \frac{k_1}{(r + \delta)q_2^1} \quad (32)$$

$$y_2 - \frac{k_2}{q^e f(q^e)} \left[ r + \delta + \left[ \frac{1 - \eta(q^e)}{\eta(q^e)} \right] (r + \delta + (1 - \delta)f(q^e)) \right] = y_1 - \frac{[r + \delta + (1 - \delta)f(q^e)]k_1}{q_2^1 f(q_2^1)} \quad (33)$$

I now show a solution to the search problem of college worker exists using two steps. In step 1, I show that there exists a solution for equations (31), (32) and (33) for a given  $\bar{c}$ . In step 2, I argue that in equilibrium, the cut-off  $\bar{c}$  must satisfies (29) and I show that a fix point exists for a function  $D(\bar{c}) = \bar{c}$ .

Fix a  $\bar{c} > 0$ . For equation (31), the right hand side converges to infinity when  $q_2^2$  converges to 0 and converges to  $k_2 + \frac{\nu(\bar{c}) - \bar{c}}{r + \delta}$ . Therefore, there always exists a solution  $q_2^2(\bar{c})$  such that equation

(31) holds. For equation (32), since  $\lambda q_2^1$  is a function of  $q^e$ , notice first that  $\frac{d(\lambda q_2^1)}{dq^e} < 0$ . To see this, differentiate  $\lambda q_2^1$  with respect to  $q^e$  and notice that

$$\frac{\partial^2 \hat{S} / \partial (q^e)^2}{\partial^2 \hat{V} / \partial (q^e)^2} > \frac{\partial \hat{S} / \partial q^e}{\partial \hat{V} / \partial q^e}$$

The left side of the inequality is greater than one, since  $\hat{S} - \hat{V}$  is a strictly convex function of  $q$ . The right side is smaller than one, since  $\hat{S} - \hat{V}$  is a strictly decreasing function of  $q$ . Hence,  $\frac{d(\lambda q_2^1)}{dq^e} < 0$ , which suggests  $q_2^1$  is a decreasing function of  $q^e$  in equation (32). The right hand side of equation (32) converges to infinity when  $q_2^1$  converges to 0 and converges to  $\frac{\nu(c)}{r + \delta}$  when  $q_2^1$  converges to infinity. Therefore, there exists a  $q_c(\bar{c}) > 0$  such that

$$\frac{y_2 - b}{r + \delta} - \frac{k_2}{\eta(q_c)q_c f(q_c)} = \frac{\nu(\bar{c})}{r + \delta}$$

and  $q_c'(\bar{c}) > 0$ . Then  $q_2^1 < \infty$  if and only if  $q^e > q_c(\bar{c})$ . For equation (33), the left hand side converges to infinity when  $q_2^1$  converges to 0 and to  $k_1$  when  $q_2^1$  converges to infinity. Define  $q_d$  such that

$$\frac{1}{r + \delta + (1 - \delta)f(q_d)} \left[ \frac{k_2(r + \delta)}{q_d f(q_d)} - (y_2 - y_1) \right] + \frac{k_2}{q_d f(q_d)} = k_1$$

and  $q_2^1 < \infty$  if and only if  $q^e < q_d$ . Notice that  $q_d < q_b$ . Define  $C_a$  and  $C_d$  such that  $q_c(C_a) = q_a$  and  $q_c(C_d) = q_d$ . If  $C_a < \bar{c} < C_d$ , the solution of the college worker's search problem is interior. If  $\bar{c} < C_a$ , the solution is associated with  $q^e = q_a$  and if  $\bar{c} > C_d$ , the solution is associated with  $q^e = q_d$ . Next, I construct a function  $D(\bar{c})$ , with  $D(\bar{c})$  equal to the right hand side of (29).  $D(0)$  satisfies

$$D(0) = \frac{k_1 \lambda q_2^1}{\eta(q_2^1)q_2^1 f(q_2^1)} - \frac{k_2}{\eta(q_2^2)q_2^2 f(q_2^2)} + \frac{y_2 - y_1}{r + \delta + (1 - \delta)f(q^e)} + \frac{k_2(1 - \delta)}{q^e} \frac{1}{r + \delta + (1 - \delta)f(q^e)}$$

and  $D(0) > 0$  given that  $q_2^2 > q_2^1$ . From equation (31), when  $\bar{c}$  increase,  $q_2^2$  decrease, therefore the first item in  $D(\bar{c})$  increases. From equation (32) and (33), when  $\bar{c}$  increases,  $q_2^1$  increases and  $q^e$  increases, therefore, the second term of  $D(\bar{c})$  decreases and  $D(\bar{c})$  is an increasing function of  $\bar{c}$ . Given that  $D(0) > 0$  and  $D(\bar{c})$  is finite (bounded by the discounted return to worker with wage equal to productivity), there exists a fixed point to  $D(\bar{c}) - \bar{c}$  with  $\bar{c} > 0$ .

Given that a solution exists to the search problem of a college worker, the solution exist to the search problem of high school worker (**P** - 4) is trivial. When the IC constraint is not binding, a high school worker faces the standard competitive search problem. When the IC constraint is binding, the equilibrium allocation of a high school worker is determined jointly by the zero-profit condition and a binding IC constraint stated in (**P** - 4).

Now I prove the first statement of Proposition 1. I first state that any equilibrium must be separating. The reason why no pooling equilibrium is as follows. Consider a contract with wage  $\omega$  in non-college job market such that both types of workers apply in equilibrium. Contract  $x$  with the corresponding queue  $q$  must lie between the zero-profit line of hiring high school workers alone and the zero-profit line of hiring college workers alone, in the space of wages and market queue as shown in Figure 2. However, at  $(q, \omega)$ , the slope of indifference curves of college workers is steeper than that of high school workers. This is because unlike college workers who search for non-college jobs because they are a stepping stone to better jobs, high school workers are more willing to remain unemployed and search for jobs offering a relatively higher wage. This means for a higher wage, high school workers are willing to accept a larger queue to be indifferent. Following the equilibrium refinement, there always exists a profitable deviation for a routine employer to post a wage higher than  $\omega$  such that only high school workers are attracted. Therefore, pooling equilibrium does not exist.

Given that any equilibrium must be separating, the rest of the proof of statement 1 follows the equilibrium definition. By definition, an equilibrium must be that workers choose where to apply to maximize expected return to search and employer make zero profits, as is characterized in problems  $(\mathbf{P} - 1)$  to  $(\mathbf{P} - 4)$ . Any equilibrium must be separating implies that the incentive compatibility constraint of a college worker must hold. This is guaranteed by the solution of problem  $(\mathbf{P} - 4)$ . Therefore, any equilibrium allocation must solve problems  $(\mathbf{P} - 1)$  to  $(\mathbf{P} - 4)$ .

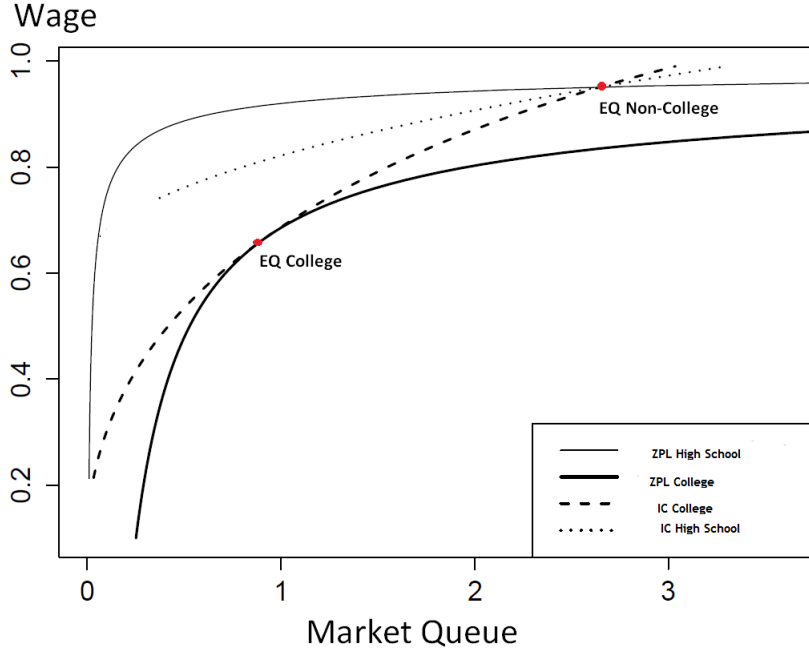
Finally, I prove the second statement of Proposition 1 hold by constructing the construction. Denote the solution to problems  $(\mathbf{P} - 1)$  to  $(\mathbf{P} - 4)$  as the follow four pairs  $(\omega_2^2, q_2^2)$ ,  $(\omega^e, q^e)$ ,  $(\omega_2^1, q_2^1)$ ,  $(\omega_1, q_1)$ . I first construct the objectives of a potential equilibrium and show that all the equilibrium conditions as defined in Definition 1 could be satisfied with these objectives.

- The set of contracts  $X^*$  includes:  $\{(\omega_2^2, 2), (b, 0)\}$ ,  $\{(\omega^e, 2), (\omega_2^1, 1)\}$ ,  $\{(\omega_1, 1), (b, 0)\}$  and  $\{(\omega_2^1, 1), (b, 0)\}$ ,
- The retention policy  $g^r$  such that for any  $s = \{(\omega, j), i\}$ ,  $s_o = \{(\omega', j'), i\}$ :

$$g^r(s, s_o) = \begin{cases} 0, & \text{if } j < j' \\ \omega' & \text{if } j \geq j' \end{cases} \quad (34)$$

- The market queue mapping  $Q$  satisfies:
  - i any wage  $\omega$  posted from a college employer attracting unemployed college workers and employed college workers,  $Q$  is such
$$k = qf(q) \frac{y_2 - \omega}{r + \delta}$$
  - ii any wage  $\omega$  posted from a non-college job employer attracting unemployed workers  $Q$  is such that

**Figure 2:** Non-College Job Markets



$$\begin{cases} k_1 = qf(q) \frac{y_1 - \omega}{r + \delta + (1 - \delta)f(q^e(\omega))}, & \text{if } \omega < \omega_2^1 \\ V_2^1 = f(q) \frac{V(\{(\omega, 1), 2\})}{1+r} + [1 - f(q)] \frac{V_2}{1+r}, & \text{if } \omega_2^1 \leq \omega \leq \omega_1 \\ k_1 = qf(q) \frac{y_1 - \omega}{r + \delta}, & \text{if } \omega > \omega_1 \end{cases} \quad (35)$$

- The conditional distribution function  $\mu$  such

- i any contract  $x$  posted from an college job employer:  $\mu[2|x] = 1$
- ii any contract  $x = \{(\omega, 1), (b, 0)\}$  posted from an non-college job employer

$$\mu[2|x] = \begin{cases} 1, & \text{if } \omega < \omega_1 \\ 0, & \text{if } \omega \geq \omega_1 \end{cases} \quad (36)$$

- The distribution of workers  $\phi = \{u_1, u_2, e_1, e_2^1, e_2^2(\omega_2^2), e_2^2(\omega^e)\}$  such that the inflow equals outflow at each state

$$\begin{aligned}
u_1 f(q_1) &= (1 - \pi - u_1) \delta \\
e_1 &= 1 - \pi - u_1 \\
u_2 [F(\bar{c}) f(q_2^2) + [1 - F(\bar{c})] f(q_2^1)] &= (\pi - u_2) \delta \\
e_2^1 [\delta + (1 - \delta) f(q^e)] &= u_2 [1 - F(\bar{c})] f(q_2^1) \\
e_2^1 (1 - \delta) f(q^e) &= \delta e_2^2(\omega^e) \\
\pi - u_2 - e_2^1 - e_2^2(\omega^e) &= e_2^2(\omega_2^2)
\end{aligned}$$

I now show that all of equilibrium conditions from Definition 1 hold. Condition (i) In all college job market, since  $Q$  traces zero profit condition, workers maximize expected return to search given  $Q$  in **(P – 1)** and **(P – 2)**. In non-college job market,  $Q$  ensures that college worker maximized given employer makes zero profits while high school worker maximized given the incentive compatibility constraint, as in **(P – 4)**. The search policy  $g$  is given by the solution of each optimization problem.

Condition (ii) The retention policy  $g^r$  maximize the continuation value of a match for employers and the zero-profit conditions satisfies (3). Condition (iii) has been shown in Lemma 1 that the zero profit condition binds for all equilibrium contracts. Condition (iv) and (v) are satisfied by the construction of  $\mu$  and  $\phi$ . Finally, the constructed belief function  $\mu$  also ensures that no pooling deviation is profitable because any deviation wage attracts at most one type of worker. *QED*

### Proof of Proposition 2

Given than 1) a solution to problems **(P – 1)** to **(P – 4)**; 2) any equilibrium must be separating and solve problems **(P – 1)** to **(P – 4)**; 3) a solution to problems **(P – 1)** to **(P – 4)** can be supported as an equilibrium, there exists a separating equilibrium. *QED*

### Proof of Proposition 3

I show that when the cost of posting college job  $k_2$  converges to zero, the IC constraint is binding. When  $k_2 \rightarrow 0$ ,  $f(q^e) = 1$  and  $\omega^e = y_2$ . A non-college job employer receives one period profit from a college worker such. The optimal search problem for a college worker is :

$$V_2^1 = \max_{\omega, q} \left\{ f(q) \frac{V(\{(\omega, 1), 2\})}{1+r} + [1 - f(q)] \frac{V(\{(b, 0), 2\})}{1+r} \right\} \quad (37)$$

s.t.

$$-k_1 + qf(q) \frac{y_1 - \omega}{1+r} \leq 0$$

where

$$V(\{(\omega, 1), 2\}) = \omega + \frac{\delta V(\{(b, 0), 2\})}{1+r} + \frac{(1 - \delta) V(\{(y_2, 2), 2\})}{1+r}$$



The F.O.C. satisfies:

$$\frac{V(\{(\omega, 1), 2\})}{1+r} - \frac{V(\{(b, 0), 2\})}{1+r} = \frac{1-\eta(q)}{\eta(q)} \frac{k_1}{qf(q)} \quad (38)$$

Substituting  $\frac{V(\{(\omega, 1), 2\})}{1+r}$  and the zero profit condition and denote  $q_0$  the solution to the problem (37), then equation (38) becomes

$$\frac{y_1}{r+\delta} - \frac{(1-\delta)y_2}{(1+r)(r+\delta)} - \frac{rV(\{(b, 0), 2\})}{(1+r)(r+\delta)} = \frac{k_1}{\eta(q_0)q_0f(q_0)} \quad (39)$$

Compare to the FOC of the high school worker's search problem where the IC constraint is absent, where  $q_1$  denote the equilibrium queue length of a high school worker's allocation

$$\frac{y_1}{r+\delta} - \frac{rV(\{(b, 0), 1\})}{(1+r)(r+\delta)} = \frac{k_1}{\eta(q_1)q_1f(q_1)} \quad (40)$$

Compare the two equations. Given that  $V(\{(b, 0), 2\}) > V(\{(b, 0), 1\})$ , and  $\eta(q)qf(q)$  is an increasing function in  $q$ ,  $q_0 > q_1$ . This implies that when  $k_2$  converges to 0, a college worker strictly prefer the undistorted allocation of a high school worker with queue length  $q_1$  to her own optimal allocation with queue length  $q_0$ .

To see this, consider the following three allocations: allocation  $(q_0, \omega_0)$  is the optimal allocation of a college worker,  $(q_1, \omega_1)$  is the undistorted optimal allocation of a high school worker as well as  $(q_0, \omega'_0)$  where  $\omega'_0$  is the wage that generates zero-profits for an employer if the corresponding queue is  $q_0$  and the worker does not search on-the-job. A college workers strictly prefer allocation  $(q_0, \omega'_0)$  to  $(q_0, \omega_0)$  because for the same matching rate the wage  $\omega'_0$  is larger than  $\omega_0$  when workers are expected to search on-the-job. A college workers also prefers allocation  $(q_1, \omega_1)$  to  $(q_0, \omega'_0)$ . This is because at  $q_0$ , employers is willing to give up more wage for an increase in matching rate when they do not expect workers to search on the job.

Finally, since a college worker prefers  $(q_0, \omega'_0)$  to  $(q_0, \omega_0)$ , the equilibrium allocation  $(q_1^*, \omega_1^*)$  of a high school worker that makes a college worker indifferent must have  $q_1^* > q_0 > q_1$ . The matching rate of a high school worker is lower than its efficient level and the unemployment rate of high school workers is inefficiently high. *QED*

#### Proof of Proposition 4

The cutoffs of post-secondary education  $\bar{e} = \frac{U_v - U_h}{\beta}$  and the cutoff of pursuing bachelor degree  $\underline{e} = \frac{U_c - U_v}{\alpha - \beta}$ . Denote  $\bar{\beta}$  such that  $\bar{e} = \underline{e}$ . At  $\bar{\beta}$ , a worker with cost  $\bar{e}$  is indifferent between obtaining a bachelor degree or a VET credential.  $\bar{\beta}$  solves:

$$\bar{\beta} = \alpha \frac{U_v - U_h}{U_c - U_h} \quad (41)$$

Given that  $U_v > U_h$  and  $U_c > U_h$ ,  $\bar{\beta} > 0$  for any  $\alpha > 0$ . *QED*

### Proof of Proposition 5

Fix  $\alpha$ . Consider  $\bar{e}(\beta) = \frac{U_v - U_h}{\beta}$  with  $\frac{d\bar{e}}{d\beta} < 0$ . Similarly,  $\underline{e}(\beta) = \frac{U_c - U_v}{\alpha - \beta}$  with  $\frac{d\underline{e}}{d\beta} > 0$ . Consider the ex-ante utility of a worker  $U_0$  as the function of  $\beta$  such that

$$U_0(\beta) = F_e(\underline{e}(\beta))U_c + [F_e(\bar{e}(\beta)) - F_e(\underline{e}(\beta))]U_v + [1 - F_e(\bar{e}(\beta))]U_h - \beta \mathbb{E}[e | \underline{e}(\beta) \leq e < \bar{e}(\beta)] - \alpha \mathbb{E}[e | e \leq \underline{e}(\beta)] \quad (42)$$

$$\frac{dU_0(\beta)}{d\beta} = - \int_{\underline{e}(\beta)}^{\bar{e}(\beta)} e f_e(e) de \quad (43)$$

The ex-ante utility of a worker in a economy without VET equals  $U_0(\bar{\beta})$  with  $\beta = \bar{\beta}$  defined in Proposition 4. At  $\beta = \bar{\beta}$ ,  $\frac{dU_0(\beta)}{d\beta} = 0$ . The ex-ante utility of a worker in a economy with VET equals  $U_0(\beta)$  with  $\beta < \bar{\beta}$ , in which case  $\frac{dU_0(\beta)}{d\beta} < 0$  and the ex-ante utility of a worker is strictly increasing as  $\beta$  decreases. By continuity,  $U_0(\bar{\beta}) < U_0(\beta)$  for any  $\beta$  such that  $0 < \beta < \bar{\beta}$ . The ex-ante utility of a worker is higher in economy with VET. Since all employers in equilibrium makes zero expected profits, the total ex-ante welfare of an economy increases by introducing VET. *QED*

## References

- ACEMOGLU, D., AND D. AUTOR (2011): “Skills, tasks and technologies: Implications for employment and earnings,” *Handbook of labor economics*, 4, 1043–1171.
- ALBRECHT, J., AND S. VROMAN (2002): “A matching model with endogenous skill requirements,” *International Economic Review*, 43(1), 283–305.
- AUTOR, D., ET AL. (2010): “The polarization of job opportunities in the US labor market: Implications for employment and earnings,” *Center for American Progress and The Hamilton Project*.
- AUTOR, D. H., D. DORN, G. H. HANSON, AND J. SONG (2014): “Trade adjustment: Worker-level evidence,” *The Quarterly Journal of Economics*, 129(4), 1799–1860.
- AUTOR, D. H., F. LEVY, AND R. J. MURNANE (2003): “The skill content of recent technological change: An empirical exploration,” *The Quarterly Journal of Economics*, 118(4), 1279–1333.
- BARNICHON, R., AND Y. ZYLBERBERG (2014): “Under-employment and the trickle-down of unemployment,” .
- BEAUDRY, P., D. A. GREEN, AND B. M. SAND (2013): “The great reversal in the demand for skill and cognitive tasks,” Discussion paper, National Bureau of Economic Research.
- CHANG, B. (2012): “Adverse selection and liquidity distortion,” *Available at SSRN 1701997*.
- CHEN, Y., M. DOYLE, AND F. M. GONZALEZ (2015): “Skill mismatch in competitive search equilibrium,” *Working Paper*.
- DELACROIX, A., AND S. SHI (2006): “Directed search on the job and the wage ladder,” *International Economic Review*, 47(2), 651–699.
- DOLADO, J. J., M. JANSEN, AND J. F. JIMENO (2009): “On-the-job search in a matching model with heterogeneous jobs and workers,” *The Economic Journal*, 119(534), 200–228.
- GAUTIER, P. A. (2002): “Unemployment and search externalities in a model with heterogeneous jobs and workers,” *Economica*, 69(273), 21–40.
- GAUTIER, P. A., G. J. VAN DEN BERG, J. C. VAN OURS, AND G. RIDDER (2002): “Worker turnover at the firm level and crowding out of lower educated workers,” *European Economic Review*, 46(3), 523–538.
- GOOS, M., A. MANNING, AND A. SALOMONS (2014): “Explaining job polarization: Routine-biased technological change and offshoring,” *The American Economic Review*, 104(8), 2509–2526.
- GUERRIERI, V., R. SHIMER, AND R. WRIGHT (2010): “Adverse selection in competitive search equilibrium,” *Econometrica*, 78(6), 1823–1862.

- KLEINER, M. M. (2005): “Reforming occupational licensing policies,” *The Hamilton Project*.
- KLEINER, M. M., AND A. B. KRUEGER (2013): “Analyzing the extent and influence of occupational licensing on the labor market,” *Journal of Labor Economics*, 31(2 pt 2).
- SAHIN, A., J. SONG, AND B. HOBIJN (2010): “The unemployment gender gap during the 2007 recession,” *Current Issues in Economics and Finance*, 16(2).
- SHI, S. (2002): “A directed search model of inequality with heterogeneous skills and skill-biased technology,” *The Review of Economic Studies*, 69(2), 467–491.
- SPENCE, M. (1973): “Job market signaling,” *The quarterly journal of Economics*, pp. 355–374.
- VEDDER, R. (2012): “Twelve inconvenient truths about American higher education. A policy paper from the center for college affordability and productivity.,” *Center for College Affordability and Productivity (NJ1)*.