A Directed Search Model of Crowding Out

Yu Chen*

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Abstract

I show that the labor-market crowding out of less-educated workers can be understood as the labor-market response to an adverse-selection problem. When high-skilled workers apply for less skill-intensive jobs, adverse selection arises when employment contracts cannot systematically discriminate against education level, even though overqualified workers are more likely to quit. In order to separate workers, the equilibrium distorts the labor-market outcomes of less-educated workers with an inefficiently high unemployment rate. Furthermore, the distortion creates a market value of post-secondary vocational education, because it acts as an entry barrier and protects less-educated workers from the competition of overqualified college graduates.

Keywords: directed search; adverse selection; crowding out

Classification: E24; J63; J64; I26

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"...hiring managers often can't understand why someone would want a lower position than what his or her background might qualify him or her for, they often assume that you're only interested in the job because you're feeling desperate. They figure you'll take it for the paycheck, but that you'll leave as soon as something more suited to your background comes along. "

—U.S. News and World Report

1 Introduction

There is a common perception that when workers are competing for the same scarce jobs, high-skilled workers displace less-educated workers by accepting less skill-intensive jobs in a phenomenon known as crowding out (e.g., Thurow (1975), Acemoglu and Autor (2011), Beaudry et al. (2013) and Barnichon and Zylberberg (2014)). However, little evidence has been found to support the view of direct crowding out. In fact, firm-level studies show that workers with higher education are not more productive at less skill-intensive jobs than less-educated workers.[1] Moreover, compared to workers whose education fits the skill requirements of their jobs, overqualified workers are found to have lower job satisfaction, to search more actively on-the-job, and to have higher quit rates, in the same jobs.[2] It is unclear why less-educated workers are the ones high-skilled workers displace, given that employers have concerns that overqualified workers are likely to use their current jobs as stepping stones towards better jobs.

This paper shows that the crowding out of less-educated workers can be understood as a labor-market response to a problem of adverse selection. This novel mechanism takes into account the interaction between the workers' job search decisions when unemployed, and their dynamic career paths. The problem arises when high-skilled workers have incentives to apply for jobs where employers anticipate filling these positions with less-educated workers, while at the same time employment contracts cannot perfectly exclude overqualified workers. As a response, the labor market equilibrium is distorted with inefficiently high unemployment rate of less-educated workers. In adopting this mechanism, this paper argues that the requirement of vocational education and regulations on licensing can help mitigate labor market inefficiencies as these are costly entry barriers for some occupations.

The study begins by analyzing a frictional labor market with on-the-job search. Search is directed in the sense that all agents take as given the trade-off, such that a job with a higher wage is associated with a lower matching rate for workers. While all workers can apply for less-skilled jobs, only college graduates can fulfill the requirements of a skilled job. The dynamic nature of the on-the-job search implies distinct career paths for workers. Unlike high school graduates, whose career ladder only go as far as less-skilled jobs, college graduates apply for these jobs as a stepping stones toward jobs that require more skills. With better on-the-job search opportunities, college graduates are willing to accept a relatively lower wage as long as the job-finding rate is sufficiently high. In contrast, high school graduates are willing to wait longer for a job that pays a relative higher wage, given that they have

[1] Gautier et al. (2002) use an administrative combined firm-worker dataset to directly test whether the quality of the workforce increases during periods of high unemployment at the firm level.

[2] Hersch (1991), Tsang et al. (1991) and Verhaest and Omey (2006) find that overqualified workers are less satisfied than well-matched workers in the same job. Wald (2005) show that overqualified workers are more active job searchers; while Tsang et al. (1991) and Maynard et al. (2006) find higher quit intentions of measured and perceived overqualified workers. Sicherman (1991), Alba-Ramirez (1993), Verhaest and Omey (2006) find that overqualified workers have significantly higher turnover rates than other comparable workers.

longer expected job tenures in less-skilled jobs.

Adverse selection arises when college graduates have incentives to apply for jobs where employers anticipate applicants who are high school graduates who will have lower job turnover. Further, when the productivity gain from hiring a worker with higher education is limited, especially for jobs where tasks can be performed by following some well-defined procedures, an overqualified worker generates lower expected profits. In this sense, as college graduates have higher expected quit rates, they are "lemons" for employers whose jobs are less skill intensive.

The key assumption in the model is that the same employer is not allowed to offer different contracts for the same job. Under fixed wage contracts, this assumption implies that any existing wage dispersion within identical jobs is caused by different paying policies across employers, a result of market segmentation. For the same job, some employers choose to pay high wages, while others offer low wages, anticipating that their posted wages will only attract certain types of workers, given that workers' job search incentives vary, based on their education, outside options, and employment status. The assumption implies that the same employer cannot discriminate qualified workers based on their educational background even though overqualified workers are relatively more likely to quit.

I show that in a competitive search equilibrium, high school and college graduates are sorted into different markets when applying for less-skilled jobs. Among these jobs, college graduates are attracted to ones that are easier to find, have higher turnover rates and pay lower wages;[3] while high school graduates prefer jobs with better compensation and lower turnover, but these types of jobs are also more difficult to obtain. This is consistent with the empirical observation that the share of college graduates are higher in the low-skilled occupations with low wages, provide little benefits, and have high turnover rates.[4] The separation, however, could be costly and generate inefficiently high unemployment for high school graduates. I show that under certain conditions, college graduates have incentives to search for the same job as high school graduates and, as a response, equilibrium contracts offered to high school graduates are distorted so that they are less attractive to college graduates. In this case, the equilibrium wage offered to high school graduates is inefficiently high and the corresponding job finding rate is inefficiently low. College graduates are discouraged from applying for these jobs because they are less willing to trade off the job-finding rate for a higher paying stepping-stone job. As a result, high school graduates are displaced from employment, even though they do not directly compete with college graduates for the same jobs.

The separation of the equilibrium relies on firms' capacity constraint to meet with workers within a period, and also whether firms can impose selective rules in hiring. Under bilateral meeting and multilateral meeting where firms do not impose any selecting rule, high school graduates and college graduates within the same market have the same chance of matching. The distribution of attracted job candidates is then important in determining firms' profits, and the equilibrium separates workers *ex-ante* into different markets. When firms meet with multiple workers and can also screen out overqualified

[3] Wages here represent the total labor income from the job that includes, for example, benefits and pensions. Jobs with a high turnover rates are often less generous in terms of these benefits.

[4] Among low-skilled occupations with high employment, the fractions of college graduates are high in occupations such as retail salespersons (24.6%), recreation attendants (23.5%) and bartenders (16.5%), while the share of college graduates who work as janitors and cleaners (5%), truck drivers (5%) and in food preparation (5.4%) are lower (Vedder et al., 2013)

workers through the hiring process such as interviews, the negative externalities caused by an additional college graduate on the matching rate of high school graduates decreases as the number of job candidate a firm can interview goes up. The equilibrium is then separating if the capacity constraint is high.

Understanding the distortion mechanism has a novel implication on the demand of education. To investigate this implication, the baseline model is extended to include educational choice. In particular, I consider the post-secondary vocational education, which primarily focuses on providing prospective workers with occupationally specific preparation. The adverse selection problem creates a demand for vocational education, which acts as an costly entry barrier and screens out college graduates by imposing the requirement of separated education. Employers wishing to fill less-skilled jobs are able to offer non-distorted contracts to workers who have obtained a vocational credential. Aside from the value of creating human capital, introducing vocational education into the labor market is welfare improving, as it offers diverse post-secondary educational choices and makes a worker *ex-ante* better off.

The idea that high-skilled workers can crowd out less-educated workers goes back to Thurow (1975), where less-educated workers are displaced out of employment when higher educated workers are ranked over less-educated workers in job competitions. In more recent work, such as Acemoglu and Autor (2011) and Beaudry et al. (2013), negative shock that directly affects high-skilled workers can crowd out less-educated workers as the high-skilled workers moved down the occupational ladder. To the sharp contrast of these competitive models, where displacement of less-educated workers are efficient skill-upgrades, the crowd-out effect in this paper is an inefficient market distortion, implying roles of labor market and educational policies for welfare improvements.

Another group of literature studies the crowding out of less-educated workers with random matching models (e.g., Albrecht and Vroman (2002) and Dolado et al. (2009)). The random matching assumption forces workers with different backgrounds compete directly for the same employers. Instead, this paper takes into account that workers with different career paths have incentive to direct their search towards employers offering different wages, for the same job. Barnichon and Zylberberg (2014) build a search model where high-skilled applicants are systematically hired over less-skilled ones. This paper, however, follows the observations that high-skilled applicants are not always preferred by employers with less-skilled jobs, and endogenously generates crowding out without imposing any ranking condition.

This paper builds on the literature that studies adverse selection in a competitive search environment. The model framework is developed from Chen et al. (2017), which extends the previous work by Guerrieri et al. (2010) with the dynamics of on-the-job search, and diverts from Chen et al. (2017) by incorporating the interaction between heterogeneous workers. While the majority of the literature studies adverse selection with unobserved productivity of workers, this paper focuses on the heterogeneity of a worker's expected job duration. Notice that even though it is contracting limitation rather than asymmetric information that prevents employers from directly separating workers, the fact that employer cannot effectively use the information leads to the same mechanism as if there is asymmetric information. Therefore, I adopt the terminology adverse selection to emphasize this mechanism, which can be applied to more general situations, where unobserved heterogeneity regarding the expected job duration affects the labor market outcomes of certain groups such as women and senior workers. Carrillo-Tudela and Kaas (2015) studies the effects of adverse selection with unobserved workers' abilities on the mobility of workers with random matching framework. While they focus on the firms' willingness to separate their applicants based on the degree of information frictions, their framework makes it difficult to address the distortion on the employment opportunities as analyzed in this paper.

The rest of the paper is organized as follows. Section 2 lays out the model. Section 3 characterizes the equilibrium with the crowding-out effect and discuss its labor market implications. Section 4 checks the robustness of the separating equilibrium under multilateral meetings. Section 5 extends the model with educational choices and discusses the implication of the crowding-out mechanism on the choice of educations. Section 6 concludes.

2 The Model

2.1 Environment

Time is discrete and continues forever. All agents are risk neutral and discount the future at a rate $r > 0$. There is a unit measure of workers, which is distinguished by their observable educational attainment. Let $i \in \{1, 2\}$ indicates these two types of workers, where type-2 workers can be interpreted as workers with bachelor's degrees and type-1 workers represent workers with only high school diplomas. The fraction of college graduates is fixed to π. In an extension in Section 5, this assumption will be relaxed with endogenous educational choices. There are also two types of jobs and $j \in \{1, 2\}$ indicates the job type. Type-2 jobs represent high-skilled jobs that involve mostly non-routine cognitive tasks, while type-1 jobs represent low-skilled jobs that mostly involve manual tasks that are both routine and non-routine. Type-2 jobs are referred to as cognitive jobs and type-1 jobs are routine jobs. Cognitive jobs require workers to have a bachelor's degree. The measure of jobs, in terms of their productivity, is determined endogenously. A routine job produces y_1. For simplicity, it is assumed that workers with college and high school education are equally productive in routine jobs. Also, if this assumption is relaxed, then the main results of this paper still hold. Further, only college graduates are productive in cognitive jobs, where output $y_2 > y_1$.

Table 1: Job Match Productivity

	Job Type	
Worker	Routine	Cognitive
High school education	y_1	0
College education	y_1	$y_2 > y_1$

Each period is divided into four stages: production, separation, search, and matching. At the beginning of a period, all workers are either employed or unemployed. Here, employed workers produce and collect wages, while unemployed workers receive benefit b. At the separation stage, an exogenous separation shock occurs with probability $\delta > 0$. Once a match is hit by a shock, the job disapears and the worker becomes unemployed and starts job searching in the next period. At the search stage, employed workers and workers who were unemployed at the beginning of the period can search for a job. Meanwhile, firms create job vacancies and state the wage contracts for each vacancy. At the matching stage, workers and employers are brought into contact to form new matches. Following Chen et al.

(2017), it is assumed that when an employed worker receives an outside offer, her current employer can choose to make a counter-offer. Then the worker decides whether to accept the offer from the poaching employer or whether to stay with her present employer. Allowing counter offers significantly reduces the dynamics and ensures the tractability of the model, since this eliminates the incentive of some employers to poach from other employers when there is no prospect of a productivity gain.

When unemployed college graduates search for jobs, they face a relative search cost $c > 0$. This cost reflects the relative difficulty of applying for a cognitive job versus a routine job. In each period a worker need to incur some costs in order to search for a cognitive job and some (different) costs for a routine job, and c reflects the difference between these search two costs. Examples of search costs include moving and traveling expenses, obtaining or renewing a license, and networking. The cost is randomly drawn from a distribution and is realized prior to the search stage, implying the existence of randomness in job search process. It also assumes that searching for a cognitive job is more costly than searching for a routine job. The purpose of having this search cost is to generate some mismatched college graduates in the model, so that I am able to study the impact of crowding-out. Without imposing this cost, all college graduates would apply to cognitive jobs when unemployed, as they are more productive jobs.[5] In equilibrium, college graduates apply for a type-1 job, when the cost realization is high, and apply for a type-2 job, otherwise. I assume that c follows an exponential distribution, $F(\cdot)$, with the parameter θ, and is independent and identically distributed across workers, and over time. The exponential distribution helps with characterizing the equilibrium as shown in the proof of Proposition 3. For employed workers, every period they received an opportunity to search on the job with probability $\lambda_e \in (0, 1]$. This probability reflects the cost of on-the-job search. The higher is λ_e, the low is the cost of on-the-job search.[6]

Let $s = \{\ell, i\} \in S$ denote the state of a type-i worker with labor-market status ℓ, and S be the set of all possible states of a worker. A worker's labor market status specifies whether the worker is employed or unemployed, and, if the worker is employed, the wage and job type. For a worker who is matched with a type-j job and is receiving wage ω, her labor-market status is a pair: $\ell = (\omega, j)$. Let $s_u = \{(b, 0), i\} \in S_u$ denote the state of an unemployed worker. Further, assume that the state of a worker is perfectly observable.

The critical assumption of the model is that contracts posted by employers cannot be conditional on the state of a worker, meaning the same employer is not allowed to pay differently based on the state of a worker for the same job. This assumption implies that any existing wage dispersion, either within identical jobs or among workers of the same type matched with the same type of jobs, is a result of market segmentation. For the same job, some employers choose to pay high wages, while others offer low wages. They need to anticipate that posting different wages might attract different workers, since workers' job search incentives vary, based on their education, outside options, and employment status.

[5]Since this paper focuses on the labor-market outcomes of less-educated workers, understanding why college graduates search for routine jobs is beyond the scope of the discussion presented here. See Chen et al. (2017) for a model with endogenous mismatch where workers apply for bad jobs with incentives.

[6]An alternative is to assume that employed college graduates face some search costs c' drawn from distribution F', similar to the unemployed workers. However, in the presence of retention offers, employed college graduates will only apply for cognitive jobs when search on-the-job. In equilibrium, they search on-the-job with a probability that is determined endogenously. Without altering the mechanism, I assume here that this probability is exogenous.

The same employer cannot discriminate qualified workers based on their educational background,[7] nor on their employment status.[8]

I focus on a labor market where each employer posts a fixed wage contract. The contracting environment is such that the employer commits to a posted wage, which remains constant until the employee receives an outside offer, in which case, the employer can choose whether to respond and offer a take-it-or-leave-it counter wage. Any renegotiation of the wage is based on mutual agreement.

The structure of the labor market is as follows. In each period, a continuum of markets may open. A market is characterized by an employment contract x and a market queue length q, which is equal to the measure of job applicants divided by the measure of job vacancies in that particular market. Markets are indexed by contracts. A contract $x = (\omega, j) \in X$ specifies the wage ω and the job type j. Notice that contracts are not conditional on the type nor the labor-market status of worker. Since workers with only high school education cannot perform cognitive jobs, it is assumed, for simplicity, that they do not apply for these jobs.

Let $Q : X \to \mathbb{R}^+$ be the queue mapping function, where X is the set of all feasible contracts. $Q(x)$ is the queue length that is associated with market x. Search is directed in the sense that both employers and workers take into account the trade-off between the wage and the queue length for any given job. Each employer can choose to post any contract, $x \in X$, with a flow cost $k > 0$. The workers observe all the posted contracts and decide where to search. Workers and firms in the same market are brought into contact by a matching technology. Matching is bilateral so that each worker meets, at most, one employer and vice versa. Workers who search in a market with market queue length q meet an employer with probability $f(q)$, and employers in the same market meet a worker with probability $qf(q)$. Following the standard search literature, it is assumed that $f(q)$ is twice differentiable, strictly decreasing and convex, with $f(0) = 1$ and $f(\infty) = 0$. It is also assumed that $qf(q)$ is strictly increasing and concave and approaches 1 as q converges to ∞. In addition, it is assumed that the elasticity of the matching function with respect to job creation $\eta(q)$, defined as $-\frac{qf'(q)}{f(q)}$ is concave, with $\eta(\infty) = 1$. This assumption is not necessary, but it simplifies the proof of the existence of an equilibrium.

It is assumed that $(r+\delta)k < y_2 - y_1$, so that the productivity gain of switching to a better job is high enough to cover the cost of hiring. This assumption is necessary to allow the possibility of job-to-job transitions The on-the-job search is a key labor-market feature of overqualified workers. It also plays an important role in the model because without the on-the-job search, an employer looking to fill a routine job is indifferent to whether she hires a worker with a college degree or only a high school diploma, and the adverse-selection problem disappears.

[7] In general, offering discriminating contracts to overqualified workers is difficult to implement. First, paying equally productivity workers differently is difficult to justify and is subject to legal risks. Also, committing *ex-ante* to not hiring overqualified workers is less credible *ex-post*. That is, upon meeting with an overqualified applicant, employers might prefer to accept the worker rather than leave the vacant job unfilled, given that searching and hiring are costly.

[8] Notice that employed and unemployed workers have incentives to self-select into segmented markets in equilibrium, which might not be the case for workers from different education groups. This is because the hiring incentives of firms are determined by the productivity of a worker as well as the expected job duration, not the employment status of a worker. Employed workers have a higher flow income, therefore would prefer to search for employers who offer higher wages while accepting lower job finding rates.

2.2 Competitive search equilibrium

Let $V(s)$ denote the discounted lifetime income of a worker in state s. A worker receives an income flow and searches for jobs. Let $U(s,c,x,q)$ denote the expected surplus from searching in the market that offers contract x and has queue length q, given the worker's current state s, where c is the realized cost of the job search, which depends on the type of job and worker. Workers direct their search among all markets to maximize their expected future income, taking as given market queue mapping Q. The value function $V(s)$ of the worker in state s satisfies the following:

$$V(s) = \omega + \frac{\delta V(s_u)}{1+r} + (1-\delta)\left\{\frac{V(s)}{1+r} + \mathbb{E}_c\left\{\max_{x\in X} U(s,c,x,Q(x))|s\right\}\right\} \tag{1}$$

for all s. The flow income is equal to $\omega = b$ if the worker is unemployed. For an employed worker, the current match is destroyed with probability δ, in which case the worker becomes unemployed. For a worker in state $s = \{\ell, i\}$, the surplus from searching is given by

$$U(s,c,x,Q(x)) = \begin{cases} -(1-i)(1-j)c + f(Q(x))\max\left\{0, \frac{V(s_o)}{1+r} - \frac{V(s)}{1+r}\right\} & \text{if } s\in S_u \\ \lambda_e f(Q(x))\max\left\{0, \frac{V(s_o)}{1+r} - \frac{V(s)}{1+r}, \frac{V(s_c)}{1+r} - \frac{V(s)}{1+r}\right\} & \text{if } s\in S\backslash S_u \end{cases} \tag{2}$$

The search cost c applies only when an unemployed college graduates searches for a cognitive job. A worker who searches in market x meets with an employer with probability $f(Q(x))$. If she fails to meet with an employer, her state is unchanged. Upon meeting with the employer, a worker accepts the offer if $V(s_o) > V(s)$, where $s_o = \{\ell', i\}$ is the worker's state after accepting the offer.

If the worker is currently employed, she receives an opportunity to search on the job with probability λ_e. If she searches in market $x = \ell'$, with $\ell' = (\omega', j')$, then the probability that she meets an employer is $f(Q(x))$. Once an outside offer is received, the current employer can respond to the worker by offering a counter-wage ω_c. If no counter-offer is made, then ω_c is equal to zero. If the worker chooses to accept the outside offer, then she starts working for the poaching employer in the next period, and her state becomes $s_o = \{\ell', i\}$. Otherwise, the worker remains matched with her current employer. Her state becomes $s_c = \{(\omega_c, j), i\}$ if the counter-offer is accepted, where j is the employer type in her current job match.

In equilibrium, workers anticipate that $\omega^c = g^r(s, s_o)$, where g_r is the firm's optimal retention policy, which is specified below. Let g be the worker's search policy and that $g(s,c)$ be the optimal search decision of a worker in state s, with cost realization c. Let $c = 0$ if the worker is employed; and g is such that

$$g(s,c) = \underset{x\in X}{\text{argmax}} U(s,c,x,Q(x)) \tag{3}$$

Also denote by g^a the acceptance policy, such that $g^a(s, s_o, s_c)$ is the probability that a worker in state s accepts the offer of contract x. Formally, g^a is such that

$$g^a(s,s_o,s_c) = \arg\max_{a\in[0,1]}\{aV(s_o) + (1-a)\max\{V(s), V(s_c)\}\} \tag{4}$$

Next, denote by $H(s)$ the sum of the discounted profits from an on-going match to the employer, where $s = \{(\omega, j), i\}$ is the state of the employee. The employer collects the profit flow and anticipates that if the match is not destroyed, then the worker will search on the job. Employers take as given the worker's optimal search policy, such that an employed worker in state $s = \{\ell, i\}$ will search in market $x' = g(s, 0)$. The probability that the worker receives an outside offer is $f(Q(x'))$. In this case, the employer decides whether to make a counter wage offer, ω_c, taking as given the worker's optimal acceptance rule. The acceptance rule specifies that the worker accepts the outside offer with probability $g^a(s, s_o, s_c)$, where s_o and s_c are the worker's states if the outside offer or the retention offer are accepted, respectively. The present value of the match for employer $H(s)$ satisfies the following:

$$H(s) = y_j - \omega + (1-\delta)\left\{f(Q(x'))\max_{\omega_c}\left\{[1 - g^a(s, s_o, s_c)]\frac{H(s_c)}{1+r}\right\} + [1 - f(Q(x'))]\frac{H(s)}{1+r}\right\} \tag{5}$$

Let $g^r(s, s_o)$ denote a solution to problem (5), the optimal retention policy g^r is contingent on the employee's current state.

Now consider the employers with unfilled vacancies. Employers choose how many vacancies to create and what type of contract to post for each vacancy. Each employer pays cost k to create a vacancy and meets with a worker with probability $Q(x)f(Q(x))$, when posting a contract $x = \ell'$. Because contracts are not conditional state of workers, employers need to form expectations about the distribution of the pool of applicants. Let $\mu(\cdot|x)$ be the conditional distribution function of the worker types who are attracted by contract x and $\mathbb{E}_s$ is the expectation with respect to $\mu(\cdot|x)$. The ex-ante return to an employer who is posting contract x is denoted as $J(x, Q(x))$ and is given by

$$J(x, Q(x), \mu(\cdot|x)) = -k + Q(x)f(Q(x))\mathbb{E}_s\left[g^a(s, s_o, s_c)\frac{H(s_o)}{1+r}\right] \tag{6}$$

where $s = \{\ell, i\}$, $s_o = \{\ell', i\}$ and $s_c = \{(g_r(s, s_o), j), i\}$. Firms take as given the workers' optimal acceptance policy $g^a(s, s_o, s_c)$, which equals the probability that contract x is accepted by a worker in state s.

Definition 1 *A stationary competitive search equilibrium consists of a set of posted contracts $X^* \subseteq X$, a set of workers' states $S^* \subseteq S$, value functions V, U, H and J and the corresponding policy functions g, g^s and g^r, a market queue mapping $Q : X \to \mathbb{R}^+$, a conditional distribution function $\mu(\cdot|x) : S \to [0, 1]$, and a distribution $\psi : S \to [0, 1]$, such that:*

(i) Workers optimize: V satisfies (1), g satisfies (3) and g^a satisfies (4). $g(s, c) \in X^*$ for all $s \in S^*$ and $S^* = \{s \in S|\psi(s) > 0\}$.

(ii) Firms optimize with free entry: $H(s)$ satisfies (5) and g^r solves (5). Moreover, for all $x \in X$, $J(x, Q(x), \mu(|x)) \leq 0$, with equality if $x \in X^*$.

(iii) Consistent beliefs : $\mu(\cdot|x)$ has support on S^*. For any $x \in X^*$,

$$\mu(s|x) = \frac{\mathbb{E}_c\left[\mathbb{1}_x\{g(s, c)\}\right]\psi(s)}{\int_S \mathbb{E}_c\left[\mathbb{1}_x\{g(s, c)\}\right]d\psi(s)}$$

for any state $s \in S$, where

$$\mathbb{1}_x\{g(s,c)\} = \begin{cases} 1 \text{ if } g(s,c) = x \\ 0 \text{ if } g(s,c) \neq x \end{cases}$$

(iv) Steady-State conditions are satisfied. For all $s \in S$

$$\int_{S^*} Pr(s_{t+1} = \tilde{s} | s_t = s) d\psi(\tilde{s}) = \int_{S^*} Pr(s_{t+1} = s | s_t = \tilde{s}) d\psi(\tilde{s})$$

where $Pr(s_{t+1}|s_t)$ is the unique distribution associated with g, g^a and g^r.

Condition (i) ensures that workers' search-and-acceptance policies are optimal for all states, taking as given the market queue length of all contracts and the employers' optimal counter-offer strategies. Condition (ii) ensures that employer's retention policies are optimal, taken as given the market queue mapping and the workers' optimal search-and-acceptance strategies. Free entry implies that equilibrium contracts generate zero expected profits for employers. Condition (iii) ensures that for contracts posted in equilibrium, the employers' beliefs about the distribution of the attracted workers are consistent with the workers' search strategies through Bayes rule. Following Chen et al. (2017), the equilibrium belief μ has full support on the equilibrium states S^*, both in- and off-equilibrium, meaning that employers do not anticipate meeting job applicants whose current wage is not offered in equilibrium. This condition is particularly important given the dynamic nature of the on-the-job search. When a firm posts a deviating contract to poach workers, it takes as given the allocation of employed workers, even though the pool of workers attracted and the associated queue length vary with the deviating contract. In other words, an out-of-equilibrium deviating poaching contract does not affect unemployed workers' job search decisions. Condition (iv) ensures the law of motion that the aggregate state of the economy is stationary, so that the inflow and outflow of workers at each state are equalized. Instead of providing a formal statement of $Pr(s_{t+1} = \tilde{s}|s_t = s)$, I provide the specific context of the equilibrium, which is characterized later in the appendix.

Equilibrium refinement

Without imposing restrictions on the belief function $\mu(\cdot|x)$, the above definition of equilibrium allows for many equilibria, each supported by a particular belief function and market queue mapping. Many of these equilibria are not of interest, for example, an arbitrary equilibrium could be supported when some markets are not open as no firms post contracts since they expect to attract no workers, while no worker applies because they anticipate high associated queues. I adopt the equilibrium refinement proposed by Chen et al. (2017), which extends the equilibrium concept proposed by Guerrieri et al. (2010) to a dynamic labor market with on-the-job search. The intuition of the refinement is closely related to the intuitive criterion presented by Cho and Kreps (1987). The refinement restricts the belief function such that positive probabilities cannot be placed on the type of worker whose payoff from applying the deviating contract is (weakly) equilibrium-dominated.

Definition 2 *A refined equilibrium is a stationary equilibrium, such that, for any $x' \notin X^*$, there does not exist a queue length $q' \in \mathbb{R}^+$ and a belief $\mu'(\cdot|x')$ on S with support on S^*, such that $J(x', q', \mu'(|x')) > 0$,* *where for any feasible* s, $\mu'(s|x') > 0$ *if and only if* $U(s, c, x', q') > U(s, c, g(s, c), Q(g(s, c)))$.

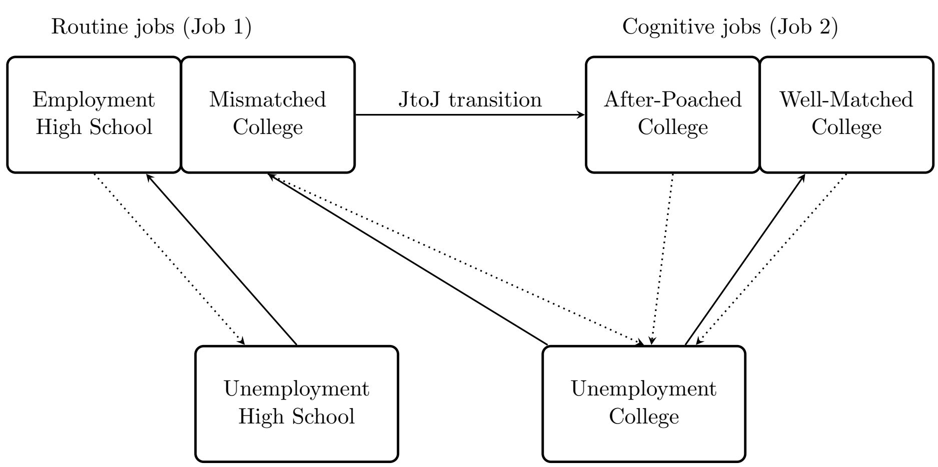
The above refinement eliminates equilibria, where some off-equilibrium contracts can generate non-negative profits for the deviating firms while providing payoffs, to the deviating workers, that are above the equilibrium level. In other words, for a refined equilibrium, there does not exist a contract that would make some workers better off while keeping all other agents not worse off, for any possible queue and any belief that places a positive probability only on workers who have a strict incentive to deviate. This refinement also excludes the equilibria where firms do not post certain contracts because they expect no worker to apply while workers do not search for these contracts because they believe the queue length associated with these contracts is too high.

3 Equilibrium with crowding out

This section analyzes the competitive search equilibrium of the above-presented model. I show that a competitive search equilibrium exists, and the equilibrium is separating with a unique allocation. A sufficient condition is provided, under which the adverse selection is present and college graduates displace high school graduates out of employment. I also discuss the model implication under different contract spaces and how it relates to other empirical observations.

Figure 1 depicts the equilibrium dynamics. In equilibrium, high school graduates search for routine jobs when unemployed. Once matched, they remain employed until their jobs are hit by an exogenous separation shock. For unemployed college graduates, some apply for cognitive jobs, while others search for routine jobs, depending on the realization of the search cost. The college graduates who are employed in routine jobs continue to search on-the-job for cognitive jobs. Once a college graduate is employed in a cognitive job, she stays with the employer until the match is exogenously destroyed.

Figure 1: labor Market Flow



The equilibrium dynamics of the labor market are straightforward since job-to-job transitions occur only when mismatched college graduates search for cognitive jobs. Following Chen et al. (2017), allowing employers to counter outside offers eliminates the complicated labor-market dynamics that are typical in models that include the on-the-job search (Delacroix and Shi, 2006). Since incumbent employers have incentives to match any outside offer, up to the value of the worker's matching productivity, equilibrium poaching wages should be no less than the value of the productivity of a routine job. With non-negative costs of posting vacancies, successful poaching happens only if there is a positive productivity gain from the new match. Given that in equilibrium there are only two types of jobs in the model—cognitive and routine jobs—poaching offers are made only from employers with vacancies in cognitive jobs who are hiring college graduates that are currently matched with employers with routine jobs. Notice that no retention happens in equilibrium, because the poaching firms can always post a wage that is slightly higher than the competing wage of the incumbent firm.

As is standard in the competitive search literature, allocation supported by a competitive search equilibrium can be characterized as the solution to a set of constrained optimization problems.[9] The basic structure of these problems is that workers maximize the expected return to their job search, subject to firms making non-negative profits. In the presence of adverse selection, this problem is also subject to the incentive compatibility constraints. Notice that the competitive search equilibrium is generally inefficient, therefore, characterizing the decentralized market rather than a representative agent's problem is essential.[10]

As shown in the equilibrium dynamics, there are four search problems there: Problem $(\mathbf{P-1})$ presents the on-the-job search decision of an employed worker. The off-job search decisions of unemployed workers who are looking for either a routine or a cognitive job are denoted as $(\mathbf{P-2})$ and $(\mathbf{P-3})$, respectively. Problem $(\mathbf{P-4})$ describes the search decision of unemployed workers with only high school education. Below, the linkage between these problems and the equilibrium is established, then the problems are presented and discussed in detail.

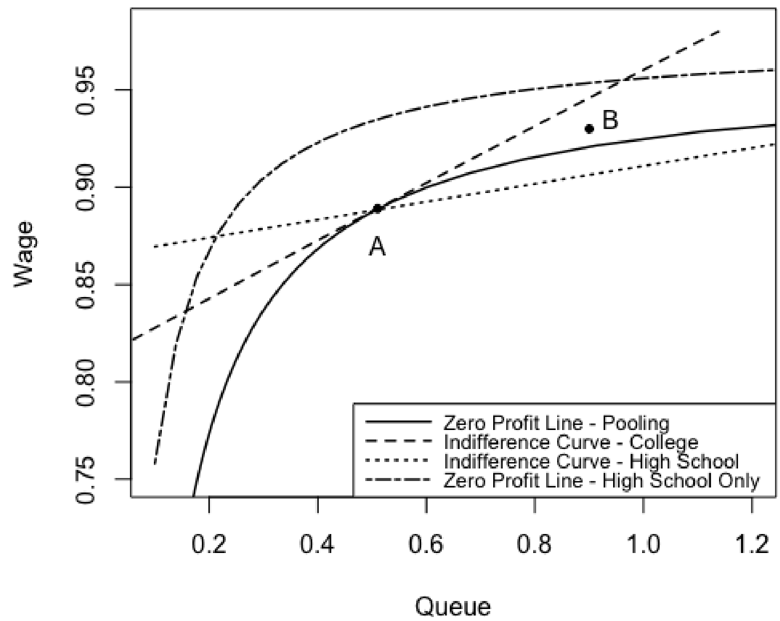
Proposition 1 *i) Any equilibrium allocation must solve problems* $(\mathbf{P-1})$ *to* $(\mathbf{P-4})$*. ii) A solution to problems* $(\mathbf{P-1})$ *to* $(\mathbf{P-4})$ *can be supported as an equilibrium.*

All formal proofs are in the appendix. I first show that any equilibrium must be separating, in the sense that college graduates and high school graduates do not compete in the same market for routine jobs, and firms anticipate to meet with only one type of worker. Figure 2 illustrates the intuition. Point A represents a hypothetical pooling equilibrium in the routine job market, where college graduates and high school graduates optimize their search given that firms make zero expected profits. The indifference curve of high school graduates crosses the indifference curve of college graduates from above since high school graduates can tolerate longer waiting time for a wage increase, keeping the expected return to search unchanged. A deviating allocation point B, which lies in the space between two indifference curves, allows firms to attract only high school graduates. Firms can be better off with the deviating

[9]See Moen (1997) for a standard competitive search equilibrium, Guerrieri et al. (2010) for a competitive search equilibrium with adverse selection, and Chen et al. (2017) has a competitive search equilibrium with adverse selection and on-the-job search.

[10]The inefficiency comes from two sources. First, workers' on-the-job search decision does not take into account incumbent firms' lifetime profit. Lack of commitment on quitting makes this model different from Menzio and Shi (2010), where contracts are assumed to be bilateral efficient. The potential adverse selection problem creates a second layer of inefficiency.

Figure 2: Pooling vs Separating in the Routine Job Market



allocation, as long as the wage is not too high (below the zero profit line that hires high school graduates only). Therefore, there is always a profitable deviation and pooling in the routine job market cannot be part of an equilibrium.

Statement i) is straightforward, given the definition of the equilibrium refinement. Since the problems are structured such that workers maximize the expected returns to a job search, given that firms make non-negative profits, any allocation that does not solve these problems cannot be supported as an equilibrium, given the existence of welfare-improving deviation. The key to showing statement ii) is to find a pair of a queue mapping and a belief function that supports the allocation as an equilibrium. Notice that such a pair is not unique. The appendix provides one example of the pairs. Combining the two statements, Proposition 1 implies that we can examine the properties of equilibrium allocation by studying the set of constrained optimization problems.

These problems are presented here. First, consider the on-the-job search problem of a college graduate receiving wage ω. Let $V(\{(\omega,1),2\})$ denote the value function of the workers, such that

$$\begin{aligned} V(\{(\omega,1),2\}) =& \omega + (1-\delta)\frac{V(\{(\omega,1),2\})}{1+r} + \delta\frac{V(\{(b,0),2\})}{1+r} \\ &+ (1-\delta)\lambda_e \max_{\omega',q} f(q)\left\{\frac{V(\{(\omega',2),2\})}{1+r} - \frac{V(\{(\omega,1),2\})}{1+r}\right\} \end{aligned} \qquad (\mathbf{P-1})$$

Subject to

$$-k + qf(q)\left(\frac{y_2-\omega'}{r+\delta}\right) \geq 0$$

$$\omega' \geq y_1$$

where the value of a match for a college graduates in a cognitive job $V(\{(\omega, 2), 2\})$ is given by

$$\frac{V(\{(\omega', 2), 2\})}{1+r} = \frac{\omega'}{r+\delta} + \frac{\delta}{r+\delta}\frac{V(\{(b, 0), 2\})}{1+r}$$

When an employed college graduate searches on the job, she chooses a pair that consists of a wage and a queue length (ω', q) to maximize the expected return to search. With probability $f(q)$, the worker meets a new employer and switches to a cognitive job (type 2 job). Otherwise, the worker stays in the current job at the same wage. The value of the new match to the worker is $V(\{(\omega', 2), 2\})$, which includes the sum of the discounted incomes flow and the value of the outside option of unemployment. Given the exogenous separation shock, the effective discount rate of a match is $r + \delta$. The first constraint of the problem states that employers should make non-negative profits. The total value of a match for the employer is $(y_2 - \omega')/(r + \delta)$, where the flow profits are discounted by the effective discount rate. The cost of hiring k is compensated by the firm's expected profits, which equals the probability $qf(q)$ of the employer meeting with a worker, times the sum of the discounted flow profits. The second constraint states that the poaching wage should be no less than the value of the worker's current matching productivity, as discussed earlier.

The solution to the above on-the-job search problem is denoted by $\{q^e(\omega), \omega^e(\omega)\}$. Notice that the optimal on-the-job search decision depends on the worker's current wage. The value function of a mismatched college graduates with wage ω can then be expressed as:

$$\frac{V(\{(\omega, 1), 2\})}{1+r} = \frac{\omega}{r+\delta} + \frac{\delta}{r+\delta}\frac{V(\{(b, 0), 2\})}{1+r} + \frac{(1-\delta)\lambda_e f(q^e(\omega))}{r+\delta}\left[\frac{V(\{(\omega^e(\omega), 2), 2\})}{1+r} - \frac{V(\{(\omega, 1), 2\})}{1+r}\right]$$

The total value of a routine job to a college graduate has two components: the value of the employment and the continuation value from the on-the-job search. The first two terms on the left-hand side of the equation present the value of the employment, including the sum of the discounted flow income and the value of the outside option of unemployment. The third term is the expected gain from the on-the-job search. With probability $(1 - \delta)\lambda_e f(q^e(\omega))$, the worker quits her job with the current employer, for a higher wage with another employer. The net gain of the job-to-job transition to the worker is $V(\{(\omega^e(\omega), 2), 2\}) - V(\{(\omega, 1), 2\})$.

Denote V_2^1 as the return to search of an unemployed college graduate who is searching for a routine job, such that

$$V_2^1 = \max_{\omega, q}\left\{f(q)\frac{V(\{(\omega, 1), 2\})}{1+r} + [1 - f(q)]\frac{V(\{(b, 0), 2\})}{1+r}\right\} \qquad (\mathbf{P-2})$$

s.t.

$$-k + qf(q)\frac{y_1 - \omega}{r+\delta+(1-\delta)f(q^e(\omega))} \leq 0$$

An unemployed college graduate searches for a pair that include a wage and a queue so as to maximize the expected return to search. With probability $f(q)$, the worker becomes employed and matches with a routine job, in the next period. Otherwise, the worker remains unemployed and searches again. This problem is subject to a non-negative profit constraint. In particular, a filled routine job is subject to both exogenous and endogenous separation. The endogenous separation arises because, once the college graduate is matched with an employer of a routine job, the worker continues to search on the job. The income flow of an employers is discounted by rate $r + \delta + (1-\delta)\lambda_e f(q^e(\omega))$, where $(1-\delta)\lambda_e f(q^e(\omega))$ is the worker's quit rate. Notice that the employer takes into account that the worker' future quit rate depends on the level of her current wage.

The third optimization problem is the problem of unemployed college graduates applying for cognitive jobs. This problem is standard, since when college graduates are matched with cognitive jobs, they do not search on the job. Denote V_2^2 the return to search when an unemployed college graduate applying for a cognitive job

$$V_2^2 = \max_{(\omega,q)} \left\{ f(q)\frac{V(\{(\omega,2),2\})}{1+r} + [1-f(q)]\frac{V(\{(b,0),2\})}{1+r} \right\} \qquad (\mathbf{P-3})$$

s.t.

$$-k + qf(q)\frac{y_2 - \omega}{r+\delta} \geq 0$$

An unemployed college graduate searches for a routine job only if the realized value of the relative searching cost is too high. Let $\bar{c}$ be the cut-off cost, such that a worker searches for a routine job if the realized cost is greater than $\bar{c}$. At the cut-off, a worker is indifferent between applying for a routine job versus searching for a cognitive job, while paying cost $\bar{c}$, such that

$$V_2^2 - \bar{c} = V_2^1 \qquad (7)$$

The value of unemployment to a college graduate $V(\{(b,0),2\})$ is then given by

$$V(\{(b,0),2\}) = b + F(\bar{c})[V_2^2 - \mathbb{E}(c|c<\bar{c})] + [1-F(\bar{c})]V_2^1 \qquad (8)$$

where $F(c)$ is the cumulative density function of c. An unemployed college graduate searches *ex-ante* for a cognitive job with probability $F(\bar{c})$, where the return to the job search is V_2^2 and the worker pays on average the expected cost $\mathbb{E}(c|c<\bar{c})$. With probability $1-F(\bar{c})$, the worker searches for a routine job with the return to the search is equal to V_2^1.

The last constrained optimization problem is the job search problem of an unemployed high school graduate. The problem is standard, except that it is subject to an incentive-compatibility constraint. Recall that employers cannot post contracts, for a routine job, that are conditional on workers' type, and college graduates have potential incentives to apply for contracts that are designed for high school graduates. Given that college graduates continue to search for jobs once they are matched with routine jobs, employers will earn negative expected profits when hiring college graduates with the same pair of wage and queue that generates zero expected profits from hiring high school graduates. Therefore, in equilibrium, no employer posts contracts aimed at hiring high school graduates that also attract college graduates. The incentive compatibility constraint specifies that offers made to high school graduates do not attract college graduates. Therefore, the value of an unemployed high school graduate is $V(\{(b,0),1\})$ such that

$$V(\{(b,0),1\}) = b + \max_{\omega,q} \left\{ f(q)\frac{V(\{(\omega,1),1\})}{1+r} + [1-f(q)]\frac{V(\{(b,0),1\})}{1+r} \right\} \qquad (\mathbf{P-4})$$

s.t.

$$-k + qf(q)\frac{y_1-\omega}{r+\delta} \leq 0$$

$$f(q)\frac{V(\{(\omega,1),2\})}{1+r} + [1-f(q)]\,\frac{V(\{(b,0),2\})}{1+r} \leq V_2^1$$

where the value of a match for a high school graduates satisfies:

$$\frac{V(\{(\omega,1),1\})}{1+r} = \frac{\omega}{r+\delta} + \frac{\delta}{r+\delta}\frac{V(\{(b,0),1\})}{1+r}$$

The incentive compatibility constraint helps us understand the displacement mechanism. The left-hand side of the equation is the expected return when a college graduate applies in market (ω, q), the same market where the high school worker applies. Notice that it includes the value of the on-the-job search in the future. The right-hand side V_2^1 is the optimal value of college graduates receive when searching for a routine job, where employers anticipate college graduates. The incentive compatibility constraint states that college graduates prefer to apply for jobs where employers expect them to search for better matches later, rather than to mimic high school graduates. In equilibrium, the constraint might or might not be binding, depending on the parameter values. When the constraint is not binding, the search decision of high school and college graduates are not related, and the allocations of high school graduates are constrained efficient. When the constraint binds, the search decision of college graduates affects the labor market outcomes of high school graduates. And the unemployment of high school graduates is distorted.

Given Proportion 1, the existence of an equilibrium relies on whether there exists a solution to problems $(\mathbf{P-1})$ to $(\mathbf{P-4})$. The following proposition establishes the existence of an equilibrium

Proposition 2 *Assume that $(y_2 - y_1)/(y_1 - b) < (1+r)/\delta$. There exists a number $\bar{k} > 0$, such that for all $k \in (0, \bar{k}]$, there is a unique equilibrium.*

The allocation supported by the equilibrium is characterized in the appendix. The idea of the proof is to use the solution of the on-the-job search problem as a mapping between the current wage and quit rate, and transform the off-job search problem of college graduates so that the problem has desired concavity properties as an optimization problem. The assumption requires that the productivity gap between the two types of jobs, relative to the match surplus of a routine job, should be bounded. This ensures routine jobs are valuable so that some college graduates would apply when they are unemployed.

In equilibrium, workers are sorted into separated markets according to their level of educational attainment. The sorting mechanism is as follows. Unemployed workers search for different markets according to their tolerance for remaining unemployed. Workers trade off matching rates and wages, based on the value of unemployment, as well as the option of conducting an on-the-job search in the future. Workers who have a higher value on unemployment are willing to wait longer for a higher wage. Meanwhile, workers with better job-switching opportunities prefer to apply for lower-wage jobs This is because the wage received out of employment counts for only a fraction of the total value of the employment. The sooner the worker becomes employed, the sooner she can start searching for a higher wage. Compared with high school graduates, college graduates have shorter expected job tenures with routine jobs, and the wages received from routine jobs count for a smaller fractions of their total expected income of employment. Therefore, college graduates are willing to trade off more wage-losses for a marginal increase in the value of the matching rate when searching for a routine job.

This separation of workers could be costly and the equilibrium would be inefficient in sorting. The proposition below shows a sufficient condition such that the incentive compatibility constraint is binding and the equilibrium suffers from adverse selection.

Proposition 3 *There exist numbers $\bar{k}_1 > 0$, $\bar{\theta} > 0$ and $\bar{y} > y_1$ such that for all $k \in (0, \bar{k}_1)$, $\theta \in (0, \bar{\theta})$ and $y_2 \in (y_1, \bar{y})$, the equilibrium suffers from adverse selection, and the unemployment rate of high school graduates is inefficiently high.*

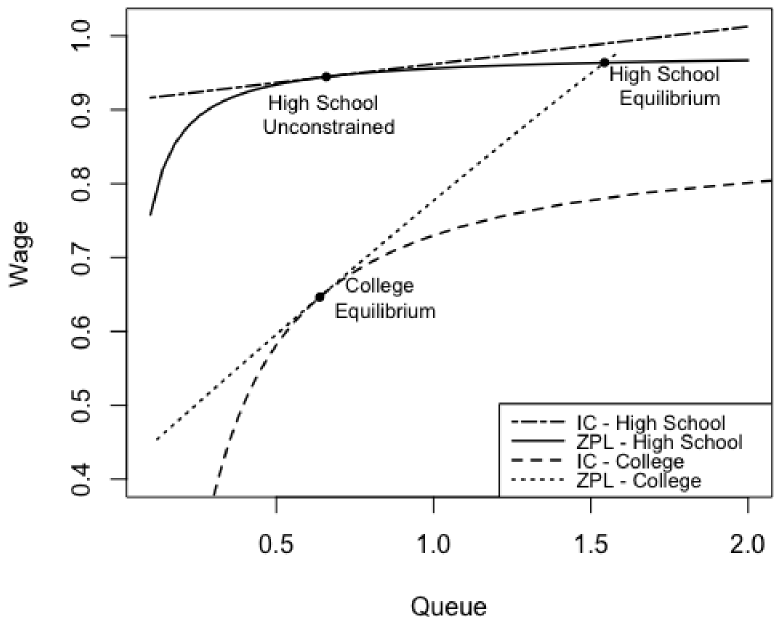
The incentive compatibility constraint binds when the cost of hiring k is sufficiently small, the productivity gap between y_2 and y_1 is sufficiently large, and the average search cost (inverse of θ) is sufficiently small. The proof starts with an environment where all college graduates search for routine jobs when unemployed (when $\theta \to 0$) and they receive all of the match surpluses after switching to a cognitive job (when the cost of posting contracts as poaching offers converges to zero). When the productivity of a cognitive job converges to the productivity of a routine job, the value of employment at a routine job is identical for all workers, even though their career dynamics are different. This is because the total surplus of matches between workers and routine jobs are the same, and the firm's share of surplus is constant. As a result, college graduates apply to markets that have the same queue length as high school graduates with a low initial wage. They have strict incentives to search in the same market as high school graduates, since they could enjoy a higher initial wage with the same matching rate and still apply for cognitive jobs later. Equilibrium with adverse selection exists for an open space with the three parameters as stated in Proposition 3.[11]

Figure 3 demonstrates the mechanism in a wage-queue space.[12] The workers' indifference curves are upward sloping, and college graduates' indifference curve cross the curve of high school graduate from below since college graduate substitute higher wages for a marginal increase in the matching rate. The zero-profit lines of employers are also upward sloping, and employers offer a lower wage to college graduates to compensate for their shorter expected job tenure at any given matching rate. Comparing the two optimal allocation for workers, where the indifference curves and zero profit lines are tangent to each other, college graduates are better-off applying for the market where high school graduates are optimized. In equilibrium, no employer enters that market as it generates negative profits for firms. Instead, equilibrium market for high school graduates is the one that makes college graduates are indifferent, which is located to the right of the initial optimal allocation. The matching rate for high school graduates is lower than the undistorted level, and their unemployment rate is inefficiently high.

[11] Numerical simulations show that the incentive-compatibility constraint binds under large ranges of reasonable parameters.

[12] The figure is simulated under parameter set: $k = 0.2$, $y_1 = 1$ $y_2 = 4$, $b = 0.4$, $\lambda_e = 1$, $r = 0.01$, $\delta = 0.1$, $\theta = 1$ and $M(u, v) = \dfrac{uv}{u + v}$.

Figure 3: Equilibrium allocation in the markets for routine jobs



The crowding out of high school graduates is a market solution to the adverse selection problem and the cost is born by high school graduates only. Their expected return from job search and the sum of discounted total labor income are inefficiently low, even though their equilibrium wages are inefficiently high. In this sense, the model implies that the skill premium, measured as the wage difference between high school and college graduates, underestimates the true returns to a college education.

One way to interpret the separating equilibrium is that the equilibrium sorts workers with different educational backgrounds into different occupations. Consider the markets within the sector of routine jobs as having different occupations with similar skill requirements and productivity. Within the routine-job sector, college graduates are attracted to occupations where jobs are easier to find, have higher turnover rates and are associated with lower wages, while high school graduates prefer to search for occupations where compensation is higher, and turnover is lower, but have more competition for becoming employed. Among low-skilled occupations with large number of employment in the labor market, the fraction of college graduates is high in occupations such as retail salespersons (24.6%), recreation attendants (23.5%) and bartenders (16.5%)[13]. Jobs in these occupations usually pay low wages and have high turnover rates. It is worth noting that the wages in the model represent the total compensation from a job. A job that provides benefits and insurance is considered as paying a higher wage than the ones that only pay a salary, even though the pay rates of the two jobs are identical. Additional compensation, such as benefits and insurance, are more attractive to high school graduates, who have limited opportunities to move up in their careers than to college graduates.

The inefficiency of displacement is in sharp contrast to the literature that studies the crowding-out

[13]See Vedder et al. (2013) for more occupations

effect using a competitive framework (for example, in Beaudry et al. (2013) and Acemoglu and Autor (2011)). In these papers, educated workers move down the job ladder, when there is a scarcity of skilled jobs, and efficiently replace less-educated workers. Their crowding-out effect relies heavily on the absolute productivity advantage of educated workers, and no externality is generated by overqualification. In contrast, this paper argues that the displacement of high school graduates is a inefficient market distortion, which does not reply on whether college graduates are more or less productive in routine jobs than high school graduates.[14] The distinction in welfare implications also lead to different policy implications. Section 5 illustrates an example on the demand of education.

3.1 Discussion

In this section, I discuss the model implications under different contract spaces, the related labor market practices, and the implication regarding the wage gaps between different education groups.

Wage-tenure Contract While the model adopts fixed wage contracts, one possible solution to the potential distortion is allowing firms to post wage-tenure contracts with back-loaded payment schemes. One could consider the optimal contract for high school graduates and an alternative contract that generates the same expected payoff but pays a lower initial wage and a higher wage later. The initial wage is less than the level of unemployment benefit and is paid for a period equals the average waiting period for cognitive jobs. This particular contract would not distort the labor market return for high school graduates, yet exclude college graduates. Therefore back-loaded wage-tenure contracts with a sufficiently low initial wage and a sufficiently steep wage increase could theoretically resolve the crowding-out problem caused by the threat of overqualified workers.

In practice, however, steeply back-loaded wage contracts are not commonly observed in the low-skilled job market. For example, study of the employers' structure opportunity in the retail industry (Waxman, 2009) finds that lower-level retail jobs are structured with relatively limited opportunities for earning increases in the absence of promotions. Taking inflation into account, the real wage gain for these employees equals only 15 cents per year, which is almost negligible given the average payment in low-skilled jobs. One reason that steeply back-loaded contract is difficult to apply is because the initial wage of low-skilled jobs is constrained by the minimal wage law, which prohibits employers from paying non-student workers too little.[15] Given that the average duration of low-skilled employment is relatively short and the average wages of these jobs are relatively low,[16] wage-tenure contracts are likely not a practical solution in the modern labor market.

Temporal versus Permanent Contract One important non-pecuniary amenity of a job is job security. In the model, all contracts are non-contingent career-long. What if employers could set the length of the contracts, for example, by making a contract either permanent or temporal? In general,

[14]The mechanism functions if college graduates are less productive than high school graduates or are more productive to a certain limit.

[15]Student-learners and full-time students working in some industries can legally be paid less than the federal minimum wage. Internship is one example.

[16]For example, the average tenure of sales clerks in retail was 6 months to 1 year (Lambert, 2008), and the median wage of sales and related occupation is 12.99 dollars in 2017, while the federal minimum hourly wage is 7.25 dollars in 2018, with the highest state minimum wage rate sitting at 11.5 dollars per hour (U.S. Bureau of Labor Statistics).

job securities in routine jobs have a higher value for high school graduates, given that they stay longer on these jobs. Therefore, the competition for permanent jobs should be more intense among high school graduates. By setting contracts with different lengths of termination, employers could separate workers if the preference for job security between high school graduates and college graduates are sufficiently different. Such a separation does not have to cause distortion when college graduates are discouraged from competition for permanent jobs with high school graduates. Studies of European countries find that in general, temporal employment has a higher rate of overeducation, and temporary contracts seem to work as effective stepping-stones towards a more stable position (Booth et al. (2002), Ortiz (2010), Mussida and Zanin (2019)), implying the possibility to separate workers using contracts with different terminations.

It is also worth noting that how workers value job securities depend on many factors, including economic conditions such as the demand for cognitive jobs and other social factors such as the welfare system. When labor markets are quite segmented, and a permanent contract is an especially valuable asset for all workers, over-education is then more likely to be found among permanent workers than among temporary ones (Ortiz, 2010). In this case, contract length alone is not enough to separate overqualified workers.

Repayment of Training Costs High turnover rate is found to be the top one concern of managers when considering overqualified workers (Maynard et al., 2009). Flight risk increases hiring costs, interrupts production, and distorts training incentives. Given that employees have legal rights to quit, there is not much room for standard contracts to punish quits. Nevertheless, for jobs that require costly on-the-job training, employees can include repayment of training cost as part of the agreement. Repayment of training costs works as a monetary penalty for voluntary quits, especially when training is job-specific and have little value for overqualified workers when moving up their careers (Robst, 1995). Although many routine jobs do not contain a high level of on-the-job training, for those positions that require long training hours with high costs, a repayment condition could discourage overqualified workers from applying.

College-High School Wage Gap Within Low-skilled Routine Jobs One implication of the model is that within low-skilled routine jobs, less-educated workers earn a higher wage than more educated workers. To see whether this implication is consistent with data evidence, I examine the skill-wage gap for different job types using the NLSY 79 dataset. Table 2 shows the percentage wage gap between workers with college education and high school graduates between year 1980 and 2000 in different job types.[17] The four job types are constructed following Yamaguchi (2012), where the skill requirements of each job are numerically measured with two dimensions, motor and cognitive tasks, given the occupation of a job.[18] Using the mean value of skill measurement for all occupations, I categorize all jobs into four groups. Jobs with simple cognitive tasks and simple motor tasks are the least skilled jobs, where the skill requirements for both dimensions are below average. Within this category, workers with a college education earn five percent less than workers who have only a high school diploma. This finding is consistent with the model prediction, where more educated workers

[17] I use 12 years of schooling as the cutoff for education groups. Sample weights apply when calculating the averages.

[18] I adopt Yamaguchi's measure of task complexity, where task data from the Dictionary of Occupation Titles is constructed into a two-dimensional vector of occupational tasks through principal component analysis (PCA).

earn a lower wage than less educated workers in low-skilled jobs that are routine based. The skill-wage gap reverses as the task complexity increases, and workers with a college education earn significantly more than high school graduates for jobs that have above-average requirements for cognitive skills.

Table 2: College-High School Wage Gap by Job Types (Percentage)

	Cognitive Simple	Cognitive Complex
Motor Simple	-4.5	28.4
Motor Complex	4.1	67.5

With some mixed results, studies of overeducation find that the wage return to overqualification/overschooling is positive,[19] meaning that overqualified workers receive higher wages than their coworkers. To understand how these empirical findings connect to this paper, it is important to identify the environment where the model applies best. First, this paper focuses on the less-skill-intensive jobs that are routine-based. The routine nature of these jobs, where job tasks are repetitive without cognitive judgments, provides little reward to cognitive skills where college graduates have an advantage. The empirical studies stated above typically look at all occupations instead of focusing on low-skilled jobs, and more importantly, they do not identify routine-based jobs. When controlling for job complexity, Gautier et al. (2002) finds that workers with many years of schooling earn less than other workers in jobs that contain simple activities and are repetitive in general, which is consistent with the model implication. The negative earning gap disappears or reverses as the job complexity increases.

Second, overqualification in this paper applies best to recent college graduates who are new entrants to the labor market. Compared with an average college graduate, new entrants have a higher chance of working in jobs that have lower skill requirements (Dudley, 2014), and are typically young workers with little labor-market experience, which negatively affects their earnings. Since empirical studies normally control for experience when estimating wage return to overeducation, it is not surprising that the lack of experience of recent college graduates also contributes to an observed lower payment compared with high school graduates in similar jobs.

Finally, while the model considers college graduates who have homogeneous ability, it is easy to apply to workers with different abilities by assuming, for example, that low-ability college graduates are the ones who apply for routine jobs when unemployed, while high-ability college graduates apply directly for cognitive jobs. In this way, the model will predict that college graduates who work in low-skilled jobs have lower ability than an average college graduate, and they earn lower wages than high school graduates in these jobs. Empirical evidence verifies a large dispersion of earnings among college graduate. In particular, Abel and Deitz (2014) show that wages for a sizeable share of college graduates below the 25th percentile are actually less than the wages earned by a typical worker with a high school diploma. The wage gap caused by the heterogeneity of workers' abilities, however, are not captured by estimating wage equations as in the overschooling literature, since a worker's ability is typically unobservable.

[19]See Leuven and Oosterbeek (2011) for a detailed survey. Studies using the IV method tend to find negative return to overqualification (Leuven and Oosterbeek, 2011).

4 Multilateral Meeting

In this section, I extend the baseline model to allow for multilateral meeting. While bilateral meeting is commonly used in competitive search models, it is useful to check whether separating workers into different markets would still be an equilibrium under multilateral meetings, a more realistic assumption for the labor market. Keeping the key assumption of single wage posting, I extend the model with two settings. Section 4.1 considers an extension of the baseline model with multilateral meeting while restricting firms to post any hiring rule. Section 4.2 further extends the settings such that firms can impose a hiring rule with a preference towards high school graduate while allowing multilateral meetings.

4.1 Multilateral meeting with no selective hiring rules

With multilateral meeting between firms and workers, each firm with a posted contract have some probabilities of meeting one or more job applicants within a period, as opposite to meeting with only one candidate under bilateral matching. When firms do not commit to any selective hiring rule, they randomly select one worker from the pool of candidates upon meeting and offer the posted contract. In this subsection, I first show that one can analyze the equilibrium using the framework developed in Section 2 with little modification, I then show that the equilibrium will always be separating under multilateral meeting without selective hiring rule and explain the intuition.

Firms' expected return of posting a contract depends on the probabilities of matching with different types of workers, which is affected by the composition of workers attracted to the market under multilateral meeting. Formally, one can rewrite the value function of firms in equation (6) as

$$J(x, Q(x), \mu(\cdot|x)) = -k + \sum_{i=1}^{2} P_i(Q(x), \mu(\cdot|x))\mathbb{E}_s^i \left[g^a(s, s_o, s_c) \frac{H(s_o)}{1+r} \right] \tag{9}$$

where $P_i(Q(x), \mu(\cdot|x))$ is the probability that a firm matches with a type i worker in market x and $\mathbb{E}_s^i$ is the expectation with respect to $\mu(\{\ell, i\}|x)$, for $i \in \{1, 2\}$. The functional form of P_i depends on the specific meeting technology. For example, with urn-ball meeting, where workers choose to apply for one firm in their desired market at random, the probability a firm matches with a high school graduate is $\sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} P_{n_1}(q_1)P_{n_2}(q_2)n_1/(n_1+n_2)$ where $q_i = \mu(\{\ell, i\}|x)q$ and $P_n(q) = e^{-q}q^n/n!$. Another example is the telephone line meeting, a case of bilateral meeting, where each worker contacts one firm and can reach the firm only if no other worker is applying at the same time. Under this meeting technology, $P_i(q, \mu) = q/(1+q)\mu(\{\ell, i\}|x)$.

When firms select workers randomly upon meeting, regardless of their types and employment status, all workers face the same probability to match in the same market. With constant return to scale matching functions, the matching rate of a worker equals $P(q)/q$ for a market with queue length q, where $P(q)$ is the overall matching rate of a firm within the market. The value functions of workers in Section 2 can be easily adjusted by replacing $f(q)$ with $P(q)/q$. Since the workers' matching rate does not depend on the distribution of applicants, they face the same search problems as in the baseline model. It is then straight forward to show that any equilibrium must be separating. Formally,

Proposition 4 *Under multilateral meeting with no hiring rule, any equilibrium must be separating.*

The proof is similar to the first part of the proof under proposition 1. Intuitively, a marginal increase in the market queue reduces the matching rate of all workers to the same amount. A marginal increase of wage, however, generates more value for a high school graduate than for a college graduate (crossing of indifference curves). In a hypothetical pooling equilibrium where all firms make zero expected profits, they must cross-subsidizing college graduates with the profits generated from high school graduates. One can always find a deviation with a marginal increase in wage and a corresponding increase in the queue such that a college graduate is indifferent from applying. This particular deviating pair will make high school graduates strictly better off, allowing firms to attract only high school graduate with positive profits, therefore breaking the refinement condition of an equilibrium.

Multilateral meeting alone does not change the separating property of the equilibrium if no particular selective hiring rule is implemented. This is not surprising as multiple meeting alters the matching rate but not the competition between workers. The negative externalities caused by overqualified workers remain the same. In the next subsection, I further extend the model with multilateral meeting and selective hiring rules that favor high school graduates. I show that whether the equilibrium is separating or pooling depends on the degree of firms' capacity on meeting with applicants.

4.2 Multilateral meeting with screening

In labor markets, firms typically receives a large number of applications per job posting, but only meet and interview a small subset before making a job offer to the most suitable candidate.[20] This meeting and screening process is different from the standard setting in the literature where meeting is either one-to-one between agents as the bilateral meeting or $n - to - 1$, where n follows a Poisson distribution as the urn-ball meetings. In this section, I adopt the framework by Cai et al. (2019) where firms can meet and interview multiple workers before making a job offer with a desirable hiring rule. I show that the equilibrium is separating when the capacity of interview is limited. In the end of this section, I briefly discuss the hiring and screening practices in the labor market.

Meeting and Hiring Process Following Cai et al. (2019), within a market, workers are randomly matched with firms[21] and firms interview applicants they meet at a random order. After every interview, and conditional on applicants remaining, the firm interviews another candidate with an exogenous probably $\sigma \in [0, 1]$, otherwise, the interviewing process stops. This general setting allows me to analyze the equilibrium under different degrees of meeting capacity of firms. When $\sigma = 0$, each firm interviews no more than one worker, and the setting converges to the baseline model with bilateral meetings. For any $\sigma > 0$, firm and workers face multilateral meetings and the number of average candidates a firm interviews increases with σ. When $\sigma = 1$, the model converges to an extreme case where firms interviews all applicants.

[20] On average, firms interview 5 to 6 candidates per vacancy (Barron et al. (1985), Blatter et al. (2012)), upon receiving dozens to hundreds of applications (Brown and Matsa (2016), Chamberlain (2015)).

[21] The microfoundation offered in Cai et al (2019) is that all agents are uniformly placed on a circle and workers send their applications clockwise to the nearest firm.

Among interviewed candidates, I allow firms to screen candidates and imposing a hiring rule such that firms give priority to high school graduates over college graduates. The hiring rule implies that while all workers have the same meeting probability within a market, their matching rates could be different. The expected return to search for workers not only depends on the market queue but also on the expected distribution of competing applicants. Therefore, I rewrite the value function of workers equation (2) such that

$$U(s,c,x,Q(x),\mu(\cdot|x)) = \begin{cases} -(1-i)(1-j)c + \frac{P_i(Q(x),\mu(\cdot|x))}{\mu(\{\ell,i\}|x)Q(x)} \max\left\{0, \frac{V(s_o)}{1+r} - \frac{V(s)}{1+r}\right\} & \text{if } s \in S_u \\ \lambda_e \frac{P_i(Q(x),\mu(\cdot|x))}{\mu(\{\ell,i\}|x)Q(x)} \max\left\{0, \frac{V(s_o)}{1+r} - \frac{V(s)}{1+r}, \frac{V(s_c)}{1+r} - \frac{V(s)}{1+r}\right\} & \text{if } s \in S \backslash S_u \end{cases} \tag{10}$$

where $\frac{P_i(Q(x),\mu(\cdot|x))}{\mu(\{\ell,i\}|x)Q(x)}$ is the probability that a type i worker is matched within a period. Similarly, the job quieting rate also depends on the expected distribution of workers' types and the present value of a match for an employer (equation (5)) is such that

$$H(s) = y_j - \omega + (1-\delta)\left\{\frac{P_i(Q(x'),\mu(\cdot|x'))}{\mu(\{\ell,i\}|x')Q(x')} \max_{\omega_c}\left\{[1-g^a(s,s_o,s_c)]\frac{H(s_c)}{1+r}\right\} + \left[1 - \frac{P_i(Q(x'),\mu(\cdot|x'))}{\mu(\{\ell,i\}|x')Q(x')}\right]\frac{H(s)}{1+r}\right\} \tag{11}$$

The above setting allows for a convenient calculation of the meeting probability which keeps the analysis tractable. Following Cai et al (2019), in a market where the queue length of all workers is q and fraction of high school graduates within all applicants is μ_1, the probability that a firm interviews at least one high school graduate equals

$$P_1(q,\mu_1) = \frac{\mu_1 q}{1 + \sigma\mu_1 q + (1-\sigma)q} \tag{12}$$

The overall matching probability of a firm, regardless of the worker's type, equals

$$P(q) = \frac{q}{1+q} \tag{13}$$

Since firms hire a college graduate only when there is no high school graduate among the interviewed candidates, the probability a firm matches with a college worker $P_2(q,\mu_1)$ then equals $P(q) - P_1(q,\mu_1)$. The matching rate for a type i worker is given by $Q_i(q,\mu_i) \equiv \frac{P_i}{\mu_i q}$. In the next proposition, I show that any equilibrium must be separating when firms are sufficiently constrained on their capacity of interviewing candidates.

Proposition 5 *There exist a $\bar{\sigma} > 0$ such for all $\sigma \in [0,\bar{\sigma})$, the equilibrium is separating.*

The intuition of the proof is as follows. When σ is small, firms face high capacity constraints and only interview a small number of applicants within a period. This means an additional college graduate graduate in the waiting queue has a higher impact on reducing the matching rate of a high school. In any pooling equilibrium, firms must cross-subsidize college graduates with high school graduates. The higher is the capacity constraint, it is easier to find a deviation that attracts high school graduates who generate more profits for firms while excluding college graduates. In other words, when firms become more constrained on the number of applicants they can meet, the matching rate of high school graduates in a pooling market is more affected by the presence of college graduates in the waiting queue, therefore, their incentives to deviate into a separating contract also increases. Overall, an equilibrium must be separating when the capacity constraint is high (when σ is low), and it becomes less binding as the capacity constraint releases.

Labor Market Practices of Screening Overqualified Workers While there is a large literature documenting the widespread phenomenon of overeducation and the consequences on the well-being of workers, empirical research on hiring practices regarding overqualification is rather limited. Surveys of hiring managers suggest that flight risk is the primary concern for employers (Bills (1992) and Maynard et al. (2009)), and the research on overqualification justifies their concerns (Skowronski, 2019). Most recruiters are willing to interview individuals whose education exceed a job's requirements; nevertheless, they tend to reject these candidates for further consideration unless they possess sufficient compensatory experience or effectively convince the recruiters that they are "intentional misfits." To screen applicants, employers relied most heavily on the personnel interview,[22] which is a costly process for recruiters. On average, firms interview only 5 to 6 candidates per vacancy (Barron et al. (1985), Blatter et al. (2012)) and employers with low skilled jobs spend even less on recruiting cost. From an empirical point of view, it is difficult to determine the cutting-off number of interviews that could change the nature of equilibrium, yet it is reasonable to believe that limitations in the capacity of interviewing candidates are applicable to most firms.

5 Implications for educational choices

This section discusses the implications of the displacement effect in the labor market on the choices and returns to education. The argument is that as an institutional solution to the adverse selection problem, post-secondary vocational education can mitigate this market distortion and it also has a higher market value than it is commonly thought.

Vocational education refers to the non-baccalaureate post-secondary education level that focuses primarily on providing training that is occupationally specific. In the US, community colleges provide most post-secondary vocational education and, once they complete their programs, students are awarded either a post-secondary certificate or an associate's degree. Also, in the US, vocational education has been growing fast over the past few decades. From 2002 to 2012, the total number of vocational-based certificates awarded per year has increased by 68 percent, which is much faster than the 39 percent in the number of bachelor's degrees awarded over the same period.[23]

In this section, the baseline model is extend to include educational choices. I show that vocational education has positive market value in terms of being an employment entry barrier to college graduates. I also argue that, in contrast to most of the literature, introducing costly entry barriers, such as vocational education and occupational licensing, is welfare improving.

[22] Only one third of employers verify education of candidates. See Moss and Tilly (2001) for a large survey of employer.
[23] National Center for Education Statistics (NCES).

5.1 A labor market with educational choices

Consider an extension of the baseline model. The distribution of workers' educational attainment is now endogenous. All workers are ex-ante identical. In period zero, an individual makes her choice of education type, based on her realized education cost, e, and the expected return to education in the labor market. An individual's education cost, e, is drawn from a distribution with the cumulative function $G(\cdot)$. This cost reflects the differences in the abilities and financial constraints that might affect one's educational development. An individual can choose either to hold only a high school diploma or to pursue post-secondary education. In the model, obtaining high school education is considered to be free, while a workers with individual cost e needs to spend αe for a college education and βe for vocational school. The total cost of education varies across individuals. Here, it is assumed that $\alpha > \beta$, such that a college education is more expensive, takes longer and requires more efforts relative to vocational education. Education is a one-time choice, accomplished before entering the labor market.[24] After period zero, all workers enter the labor market with either a high school diploma, a vocational credential or a bachelor's degree.

Now there are two types of jobs that require less skill: the ones that have no educational requirement (referred to as routine jobs) and the ones that require a vocational credential (referred to as vocational jobs and denoted as v), as well as cognitive jobs in the labor market. The cognitive jobs and routine jobs are the same as in the baseline model. It is assumed that vocational jobs and routine jobs are equally productive, such that $y_2 > y_1 = y_v$. This assumption is made to isolate the role of vocational education in mitigating the market distortion in terms of its value of providing human capital.

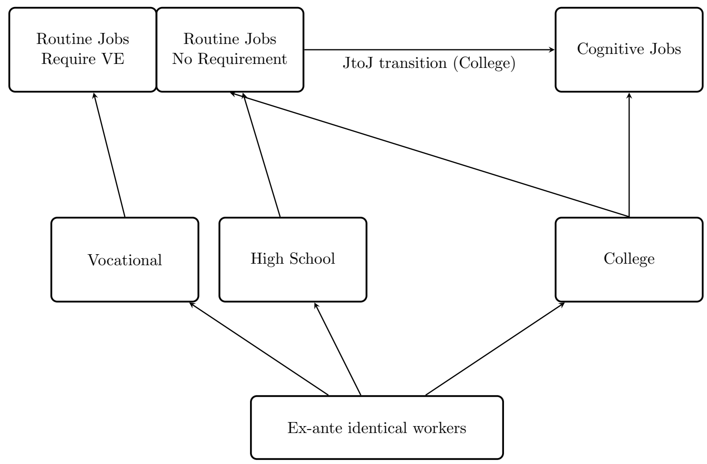
The labor market dynamics are depicted in Figure 4. The flow of high school graduates and college graduates are identical as in the baseline model. *Ex-post*, high school and college graduates' job-search options are not affected by the presence of vocational jobs since there are entry barriers to vocational jobs. An unemployed worker with a vocational credential can apply for either a routine job or a vocational job. In equilibrium, the worker only applies for vocational jobs, and once matched, does not search on the jobs. To see this, notice that while the productivity of both jobs are equal, routine jobs might suffer from market distortions. Therefore, the return to a job search when applying for a vocational job weakly dominates the return to a job search for a routine job. Further, workers with vocational education have no incentive to mimic high school graduates and search on the job as do college graduates. No firm with vocational-job vacancies has the incentive to poach workers from routine jobs, given that the productivity of workers in routine jobs and vocational jobs is equal.[25] For the same reason, once matched with a vocational job, there is no value to be gained for searching on the job.

Denote by U_h, U_v, U_c the equilibrium returns to a job search from unemployment for a high school graduate, a worker with a vocational credential and a worker with a bachelor's degree, respectively.

[24] While the model restricts workers from entering both university and vocational education, in reality, the fraction of workers with both credentials is small. For example, Ferrer and Riddell (2002) find that workers with both a bachelor's degree and a vocational credential accounts only for 3.4% of all full-year full-time workers, using 1996 Census Canada data.

[25] If the productivity of a vocational job is greater than that of a routine job, then workers with vocational education might have an incentive to mimic high school graduates. However, when high school graduates are displaced due to the incentives that encourage college-educated workers to search for these types of jobs, workers with vocational education do not further distort the market. Therefore, there is no incentive for workers with vocational education to mimic high school graduates.

Figure 4: Labor market flows with educational choices



Since college graduates cannot apply for vocational jobs, the equilibrium allocation of workers with vocational education is the constrained efficient allocation that a high school graduate would receive if there is no displacement.

A worker's educational choice before entering the labor market is determined as follows. Let $\bar{e}$ be the cut-off such that a worker with the an educational cost that is greater than $\bar{e}$ does not pursue post-secondary education. The return to vocational education, relative to that of a high school diploma, must be equal to the cost of the vocational education with $\bar{e}$, such that

$$U_h = U_v - \beta\bar{e} \tag{14}$$

Similarly, let $\underline{e}$ be the cut-off such that a worker with an educational cost that is lower than $\underline{e}$ enters university to obtain a bachelor's degree. The cut-off $\underline{e}$ must satisfy the following:

$$U_v - \beta\underline{e} = U_c - \alpha\underline{e} \tag{15}$$

The demand for vocational education is positive if and only if $\bar{e} > \underline{e}$.

Demand for vocational education When the economy suffers from adverse selection, the market for routine jobs is distorted, and the labor-market return to the high school graduate is inefficiently low. Unlike routine jobs, there are barriers to entry for jobs that require vocational education. Therefore, an employer that is offering vocational jobs can exclude college graduates from applying and post non-distorted wages. By obtaining vocational education, workers gain access to markets that do not suffer from the adverse selection and can improve their labor market outcomes. The value of vocational education, as an entry barrier, increases under the existence of the displacement that distorts the labor market.

This idea is formalized in the next proposition. As long as the cost of vocational education is not too high, relative to college education, there exists a demand for vocational education when the labor market suffers from adverse selection.

Proposition 6 *When an equilibrium exists with adverse selection, there is a level $\bar{\beta} \in (0, \alpha)$ such that for any $\beta \in (0, \bar{\beta})$, the equilibrium has positive demand for vocational education.*

The gain from vocational education is equal to the level of distortion of the labor-market return to high school graduates that is caused by the threat of overqualified college graduates. The higher the level of this distortion, the higher is the return to vocational education. As a barrier to entry, vocational education mitigates adverse selection by excluding college graduates from applying for less-skilled jobs. In this sense, vocational education helps sort to the right workers jobs that require less skill. When the cost of vocational education does not exceed the gain, individuals with cost realizations between $\underline{e}$ and $\bar{e}$ find it optimal to pursue vocational education.

When there exists a demand for vocational education, an individual enters college with probability $G(\underline{e})$, chooses vocational education with probability $G(\bar{e}) - G(\underline{e})$, and graduates with a high school diploma only, with probability $1 - G(\bar{e})$. Let U_0 be the ex-ante utility of a worker in period 0, before the realization of the individual's educational cost. The level of U_0 is equal to the expected return from the labor market, net the expected costs paid to acquire the education, such that

$$U_0 = G(\underline{e})U_c + [G(\bar{e}) - G(\underline{e})]U_v + [1 - G(\bar{e})]U_h - \beta\mathbb{E}[e|\underline{e} < e \leq \bar{e}] - \alpha\mathbb{E}[e|e \leq \underline{e}]$$

Since all workers are identical ex-ante and all employers in equilibrium make zero expected profits, the total welfare of different economies can be ranked by comparing the levels of U_0. Comparing with an economic where the option for post-secondary education is college education only (with the same education cost rate α), the following proposition shows that, costly vocational education can improve welfare, when the labor market suffers from adverse selection.

Proposition 7 *When an equilibrium exists with adverse selection, the option to undertake vocational education makes workers better off ex-ante, for $\beta \in (0, \bar{\beta})$.*

Compared with the economy where vocational education is not available, there is a lower fraction of both high school graduates and college graduates in the economy that has a positive demand for vocational education. Vocational education attracts some high school graduates since their vocational credentials allow them the search in markets that are not distorted by college graduates. It also attract some college graduates whose cost of post-secondary education is high. When the adverse selection problem is present, introducing vocational education to the labor market strictly improves welfare, even without considering a productivity premium, as long as vocational education is not too costly. Again, this result is different from the literature on competitive labor markets, where a barrier to entry creates monopoly power, distorts efficiency and reduce the overall welfare of the economy. This paper shows instead that when the displacement effect occurs through adverse selection, vocational education as a barrier to entry has an important benefit to the labor market, as it improves welfare by sorting the right workers into the right markets.

Occupational licensing The above-discussed mechanism also helps understand the market value of occupational licensing. Occupational licensing is often considered as a barrier that restricts entry to certain professions. According to Kleiner and Krueger (2013), nearly 30 percent of US workers with more than a high school education, but not a bachelor's degree, were required to hold a license for their jobs. To obtain a license, candidates need to pay a series of fees and pass exams, which usually takes weeks and sometimes several months. One concern of occupational licensing is that as an entry barrier, it may cause job losses by increasing employment costs (Kleiner, 2005). This concern is particularity relevant for occupations that are licensed for conformance, which consists a large group of low- to intermediate-skilled occupations, where obtaining a license does not require long periods of training and skills development.[26] Conformance licensing places little emphasis on the skills development and does not create an occupational monopolies (Cooney, 2013), therefore, the monetary and time investment required to obtain a license incurs a costly barrier to entry in these occupations. This paper shows that by imposing an employment entry cost, occupational licensing can have an important benefit to employment as it provides protections for middle- and low-skilled workers from competition from over-qualified college graduates. Imposing occupational licensing, in certain occupations, can not only increase the employment opportunities of low- to middle-skilled workers, but also increase labor-market efficiency.

6 Conclusion

This paper presents a model of a frictional labor market where both high-skilled and less-educated workers can apply for less skill-intensive jobs. The study argues that an adverse selection problem arises, when employment contracts cannot perfectly exclude overqualified workers, even though they have high quitting rates in less-skilled jobs. The study also shows that less-educated workers can be displaced out of employment when employers have concerns that overqualified workers will use less skill-intensive jobs as stepping stones towards better jobs. As a response to adverse selection, the labor market generates inefficiently high unemployment among less-skilled workers as a mechanism to separate high-skilled workers who are searching for stepping-stone opportunities. The separating equilibrium exists under both bilateral and multilateral meeting when firms' capacity of meeting with many job candidates is limited.

By extending the model to include educational choices, this study shows that the displacement caused by adverse selection creates a demand for vocational education. As a costly entry barrier, employers can screen out overqualified workers from seeking stepping-stone jobs by imposing requirements of vocational credentials and occupational licensing. Vocational education and licensing regulations can help mitigate the labor-market inefficiency caused by the adverse selection and protect low- to middle-skilled workers from the competing against overqualified high-skilled workers in the same job market.

[26] Examples include transport drivers such as truck drivers, bus drivers, childcare workers, security personnel, miners, earth-moving plant operators, and controlled personal service workers.

Appendix

Proof of Proposition 1

This proof contains three parts. First, I will show that an equilibrium must be separating, meaning that high school graduates and college graduates search for different wages when applying for routine jobs. In other words, there does not exist a pooling equilibrium where college and high school graduates search in the same market for routine jobs. Second, I will show that condition $i)$ and $ii)$ must hold for a separating equilibrium.

Consider a pooling equilibrium, such that college and high school graduates search in the same market (ω_p, q_p) for routine jobs. Employers with routine jobs anticipate meeting with both types of workers with a fraction π_p of college graduates. In this hypothetical equilibrium, the expected search return for a routine job employer is

$$-k + q_p f(q_p)\left[\pi_p \frac{y_1 - \omega_p}{r + \delta + (1-\delta)\lambda_e f(q^e(\omega_p))} + (1-\pi_p)\frac{y_1 - \omega_p}{r+\delta}\right] = 0$$

where $q^e(\omega)$ is the optimal on-the-job search decision, defined as the solution of problem $\mathbf{P-1}$. Firms make zero expected profits, which implies they hire high school graduates to cross-subsidize the hiring of college graduates, such that

$$-k + q_p f(q_p)\left[\frac{y_1 - \omega_p}{r+\delta}\right] > 0$$

I show that there always exists a deviating contract $x' = \{(\omega', 1), (b, 0)\}$, a queue length q' and a belief function $\mu(\{(b,0),1\}|x') = 1$, such that the high school graduates are strictly better-off applying to x', while employers make positive profits posting x'. In other words, there always exists a deviation such that the definition refinement is violated.

Let $\omega' = \omega_p + \epsilon$. Let q_1' be the queue length so that (ω', q_1') generates the same return as (ω_p, q_p) for high school graduates. q_1' must satisfy the following condition

$$f(q_p)\left[V(\{(\omega_p,1),1\}) - V(\{(b,0),1)\})\right] = f(q_1')\left[V(\{(\omega',1),1\}) - V(\{(b,0),1)\})\right] \tag{16}$$

Similarly, let q_2' be the queue length such that (ω', q_2') generates the same return as (ω_p, q_p) for college graduates. q_2' must satisfy

$$f(q_p)\left[V(\{(\omega_p,1),2\}) - V(\{(b,0),2)\})\right] = f(q_2')\left[V(\{(\omega',2),1\}) - V(\{(b,0),1)\})\right] \tag{17}$$

Compare q_1' and q_2'. Since $\partial V(\{(\omega,1),1\})/\partial\omega = 1/(r+\delta)$, while $\partial V(\{(\omega,1),2\})/\partial\omega = 1/[r+\delta+(1-\delta)f(q^e(\omega))] < 1/(r+\delta)$, for equations (16) and (17) to hold, q_1' must be larger than q_2'.

Let $\underline{q}_1'$ be the queue length such that firms earn zero profits hiring high school graduates with ω'. $\underline{q}_1'$ follows

$$-k + \underline{q}_1' f(\underline{q}_1')\left[\frac{y_1 - \omega'}{r+\delta}\right] = 0$$

Notice that q_1' increases with ϵ, and $q_1' \to q_p$ as $\epsilon \to 0$. At (ω_p, q_p), firms earn a positive profit hiring high school graduates. Therefore, there always exists an $\epsilon > 0$ such that $q_1' > \underline{q}_1'$.

Now, I can construct the deviation. A deviating contract $x' = \{(\omega', 1), (b, 0)\}$ together with a queue length $q' \in (\underline{q}_1', q_1')$ and a belief function with $\mu(\{(b, 0), 1\}|x') = 1$, attract only high school graduates to apply. The expected return to search for a routine job is higher than the equilibrium level for high school graduates because $q' < q_1'$, and employers posting x' earn a higher expected return than in the equilibrium because $q' > \underline{q}_1'$. Therefore, (ω_p, q_p) cannot be an equilibrium allocation.

Next, I will focus on a separating equilibrium. To show the statement i), consider an allocation that does not solve problems $(\mathbf{P-1})$ to $(\mathbf{P-4})$. Then there must exist a deviating contract and a market queue, constructed from the solution of problems $(\mathbf{P-1})$ to $(\mathbf{P-4})$, such that the posted contract generates non-negative profits for firms, while some workers are strictly better off. This violates the definition of a refined equilibrium. Therefore, any allocation that does not solve problems $(\mathbf{P-1})$ to $(\mathbf{P-4})$ cannot be an allocation in equilibrium. Statement i) must be true.

To illustrate statement ii), suppose that a solution to problem $(\mathbf{P-1})$ to $(\mathbf{P-4})$ exists. It is straight forward to construct the equilibrium contracts X^* and equilibrium states S^* based on the the optimal wages from the solution. To construct a queue mapping Q^* and a belief function $\mu(|x)$ that supports the equilibrium, consider the following example. For the cognitive-job sector, the belief is degenerated such that firms expect to meet only college graduates. The queue mapping is such that firms earn zero expected profits. For the routine-job sector, firms expect to meet with college graduates only for the posted wages that are lower than the optimal wage of high school graduates; they also expect to meet with high school graduates only for posted wages that are equal to or higher than the optimal wage of high school graduates. The Q^* is such that firms makes zero expected profits for any wage less than or equal to the optimal wage of college graduates and any wage higher than the optimal wage of highs school workers. For markets with wages in between the above ranges, the corresponding queue length is fixed at the level of the optimal queue that solves the high school graduates' problem. This particular queue mapping and belief function satisfy the equilibrium conditions (i) to (iii), such that workers optimize given Q^*, firms optimize given Q^* and $\mu(|x)$ and earn zero profits at S^*. The belief in equilibrium is consistent with the actual distribution. Finally, using the optimal market queues (job arrival rates) from the solution, one can construct the stationary distribution of workers by equating the inflow and outflow of equilibrium states as stated in condition iv.

Proof of Proposition 2

Given *Proposition 1*, it is sufficient to show that a solution exists to problems $(\mathbf{P-1})$ to $(\mathbf{P-4})$. Also, notice that the search problems of college graduates are independent from the search problems of the high school graduates. Therefore, I first show that a solution exists to the problems $(\mathbf{P-1})$ to $(\mathbf{P-3})$ of college graduates, and then show that there is always a solution that solves problem $(\mathbf{P-4})$ of high school graduates.

College graduates

Consider first the on-the-job search problem $(\mathbf{P-3})$. The first order conditions for an interior solution are given by

$$\lambda q = 1$$

where λ is the relevant Lagrange multiplier and

$$\frac{V(\{(\omega',2),2\})}{1+r} - \frac{V(\{(\omega,1),2\})}{1+r} = \lambda q \frac{1-\eta(q)}{\eta(q)} \frac{y_2-\omega'}{r+\delta}$$

denote $\{\omega^e(\omega), q^e(\omega)\}$ a solution to the problem.

Lemma 1 *For any* $\omega \in [0,\, y_1]$*,* $\{\omega^e(\omega), q^e(\omega)\}$ *is given uniquely by the pair* (ω', q) *with* $y_1 \leq \omega' < y_2$ *and* $0 < q_a \leq q' \leq q_b < \infty$ *such that*

$$q' f(q') \left(\frac{y_2 - \omega'}{r+\delta} \right) = k$$

$$\frac{\omega' - \omega}{r+\delta+(1-\delta)\lambda_e f(q')} \geq \frac{1-\eta(q)}{\eta(q)} \frac{k}{q' f(q')}$$

and $q' \geq q_a$ *with complementary slackness, where* q_a *is given by*

$$q_a f(q_a) \left(\frac{y_2 - y_1}{r+\delta} \right) = k$$

and $q_b > q_a$ *is given by*

$$\frac{y_2-y_1}{r+\delta} = \frac{k}{q_b f(q_b)} \left(1 + \left(\frac{1-\eta(q_b)}{\eta(q_b)} \right) \left(\frac{r+\delta+(1-\delta)\lambda_e f(q_b)}{r+\delta} \right) \right)$$

Proof: the first condition, that is stated in the lemma, comes from the non-negative profit constraint. The second condition follows from combining the two first order conditions, substituting $V(\{(\omega,1),1\})$ and $V(\{(\omega',2),2\})$ with the zero-profit constraint, such that

$$\frac{V(\{(\omega',2),2\})}{1+r} - \frac{V(\{(\omega,1),2\})}{1+r} = \frac{\omega' - \omega}{r+\delta+(1-\delta)\lambda_e f(q')}$$

The solution is interior, i.e. $\omega' \geq y_1$ if and only if $q' \geq q_a$. Notice that the assumption $(r+\delta)k < y_2 - y_1$ ensures that $q_a < \infty$. Combining the two conditions in the lemma implies

$$y_2 - \omega - \frac{k}{q' f(q')} \left(r+\delta + \frac{1-\eta(q')}{\eta(q')} \left[r+\delta+(1-\delta)\lambda_e f(q') \right] \right) = 0$$

where q' increases monotonically with ω given that $qf(q)$ and $\eta(q)$ are increasing functions while $f(q)$ is a decreasing function. q_b is defined such that $\omega = y_1$. The interior solution $q^e(\omega)$ is the unique value of q' that solves the above equation with $q_a \leq q' \leq q_b$. It is clear that $\infty > q_b > q_a$. *QED*

Invert the above equation such that the current wage ω is a function of the future q'

$$W(q') = y_2 - \frac{k}{q' f(q')} \left(r+\delta + \frac{1-\eta(q')}{\eta(q)} \left[r+\delta+(1-\delta)\lambda_e f(q') \right] \right)$$

for all $q_a \leq q \leq q_b$. Substituting $W(q')$ into $V(\{(\omega,1),2\})$, one can express the value of a routine job for college graduates as a function of q'

$$\frac{\hat{V}(q')}{1+r} \equiv \frac{V(\{(W(q'),1),2\})}{1+r} = \frac{y_2}{r+\delta} - \frac{k}{q' f(q') \eta(q')} + \frac{\delta}{r+\delta} \frac{V(\{(b,0),2\})}{1+r}$$

Note that

$$\frac{d}{dq}\left(\frac{\hat{V}(q)}{1+r}\right) = \frac{k}{qf(q)}\left(\frac{\eta'(q)}{\eta^2(q)} + \frac{1}{q}\left(\frac{1-\eta(q)}{\eta(q)}\right)\right)$$

which is positive for any $q \in [q_a, q_b]$ given that $\eta(q) < 1$ and $\eta'(q) > 0$, and decreasing if $\eta'(q)/\eta^2(q)$ is a decreasing function, which is given by the concavity assumption of η. Therefore $\hat{V}(q)$ is a strictly concave function of q.

Let $S(s)$ be the net surplus of a match, denoted by the state s of the worker employed in the match. Define $\hat{S}(q')$ such that

$$\frac{\hat{S}(q)}{1+r} \equiv \frac{S(\{(W(q),1),2\})}{1+r} = \frac{y_1 - W(q)}{r+\delta+(1-\delta)\lambda_e f(q)} + \frac{V(\{(W(q),1),2\})}{1+r} - \frac{V(\{(b,0),2\})}{1+r}$$

The first term to the right of the second equation is the firm's share of surplus and the two remaining terms add up to the worker's share of surplus.

Lemma 2 $S(\{(W(q),1),2\}) - V(\{(W(q),1),2\})$ *is a strictly decreasing and convex function of* $q \in [q_a, q_b]$. $S(\{(W(q),1),2\}$ *is a concave function of* $q \in [q_a, q_b]$ *and is maximized at* $q = q_b$ *.*

Proof:

$$\begin{aligned}\frac{d}{dq}\left(\frac{\hat{S}(q)}{1+r} - \frac{\hat{V}(q)}{1+r}\right) &= \frac{(1-\delta)\lambda_e f'(q)}{[r+\delta+(1-\delta)\lambda_e f(q)]^2}\left(y_2 - y_1 - \frac{k(r+\delta)}{qf(q)}\right) \\ &\quad - \frac{k}{qf(q)}\left[\frac{1-\eta(q)}{q}\left(\frac{1-\eta(q)}{\eta(q)} + \frac{r+\delta}{r+\delta+(1-\delta)\lambda_e f(q)}\right) + \frac{\eta'(q)}{\eta^2(q)}\right]\end{aligned}$$

The first term to the right of the equation is non-positive, since $f'(q) < 0$ and $qf(q)(y_2 - y_1) \geq k(r+\delta)$ for $q \geq q_a$. The second line is negative given $\eta(q) < 1$ and $\eta'(q) > 0$. Hence, $\hat{S}(q) - \hat{V}(q)$ is a strictly decreasing function of q for $q \geq q_a$. Also, the first term to the right of the equation is increasing, given that $f(q)$ is decreasing and convex and $qf(q)$ is increasing. The second line is increasing since $\eta'(q)/\eta^2(q)$ is decreasing with q. Therefore, $\hat{S}(q) - \hat{V}(q)$ is strictly convex.

To show that $S(\{(W(q),1),2\}$ is concave, differentiate $S(\{(W(q),1),2\}$, which gives

$$\frac{d}{dq}\left(\frac{\hat{S}(q)}{1+r}\right) = \frac{(1-\delta)\lambda_e}{q^2[r+\delta+(1-\delta)\lambda_e f(q)]^2}\left(k(1-\eta(q)) - \frac{(r+\delta)\eta(q)}{r+\delta+(1-\delta)\lambda_e f(q)}\left[qf(q)\frac{y_2-y_1}{r+\delta} - k\right]\right)$$

the right-hand side of the equation is strictly decreasing in q. It equals to zero if

$$\frac{y_2 - y_1}{r+\delta} = \frac{k}{qf(q)}\left(1 + \left(\frac{1-\eta(q)}{\eta(q)}\right)\left(\frac{r+\delta+(1-\delta)\lambda_e f(q)}{r+\delta}\right)\right)$$

which holds at $q = q_b$. *QED*

Now consider the off-job search problem ($\mathbf{P-2}$). The objective function in ($\mathbf{P-2}$) is not generally concave in $\{\omega, q\}$, because the current wage affects workers' future quit rates (q^e). Instead of solving the original problem, consider the following transformed problem

$$V_2^1 = \max_{q',q}\left\{f(q)\frac{V(\{(W(q'),1),2\})}{1+r} + [1-f(q)]\frac{V(\{(b,0),2\})}{1+r}\right\} \qquad (\mathbf{P-2'})$$

s.t.

$$-k+qf(q)\left[\frac{S(\{(W(q'),1),2\})}{1+r}-\frac{V(\{(W(q'),1),2\})}{1+r}+\frac{V(\{(b,0),2\})}{1+r}\right]\leq 0$$

$$q_a \leq q'$$

The first order conditions for an interior solution of problem ($\mathbf{P-2'}$) are given by

$$\frac{V(\{(W(q'),1),2\})}{1+r}-\frac{V(\{(b,0),2\})}{1+r}=\lambda q\left[\frac{1-\eta(q)}{\eta(q)}\right]\frac{k}{qf(q)}$$

and

$$\lambda q=-\frac{\frac{d}{dq'}\left(\frac{V(\{(W(q'),1),2\})}{1+r}\right)}{\frac{d}{dq'}\left(\frac{S(\{(W(q'),1),2\})}{1+r}-\frac{V(\{(W(q'),1),2\})}{1+r}+\frac{V(\{(b,0),2\})}{1+r}\right)} \tag{18}$$

Given that $V(\{(W(q'),1),2\}$ and $V(\{(W(q'),1),2\}$ are functions of q', λq can be expressed as a function of q'. Substituting $V(\{(W(q'),1),2\})$ into the first order condition:

$$\frac{y_2}{r+\delta}-\frac{k}{q'f(q')\eta(q')}-\frac{r}{r+\delta}\frac{V(\{(b,0),2\})}{1+r}=\lambda q\left[\frac{1-\eta(q)}{\eta(q)}\right]\frac{k}{qf(q)}$$

Denote (q^e,q_2^1) the solution to problem ($\mathbf{P-2'}$), the value of unemployment $V(\{(b,0),2\})$ can be expressed using the above equation as

$$\frac{V(\{(b,0),2\})}{1+r}=\frac{1}{r}\left(y_2-\frac{k(r+\delta)}{q^ef(q^e)\eta(q^e)}-\lambda q\left[\frac{1-\eta(q_2^1)}{\eta(q_2^1)}\right]\frac{k(r+\delta)}{q_2^1f(q_2^1)}\right) \tag{19}$$

Substituting this back to the first order condition, the optimal value of the job search return for a routine jobs V_2^1 for a college graduate is

$$V_2^1=\lambda q_2^1\left(\frac{1-\eta(q_2^1)}{\eta(q_2^1)}\right)\frac{k}{q_2^1}+\frac{1}{r}\left(y_2-\frac{k(r+\delta)}{q^ef(q^e)\eta(q^e)}-\lambda q_2^1\left[\frac{1-\eta(q_2^1)}{\eta(q_2^1)}\right]\frac{k(r+\delta)}{q_2^1f(q_2^1)}\right) \tag{20}$$

Consider the last problem of a college graduate, i.e. the off-job search problem ($\mathbf{P-3}$). The relevant first order conditions for an interior solution of the problems are

$$\lambda q=1$$

and

$$\frac{V(\{(\omega,2),2\})}{1+r}-\frac{V(\{(b,0),2\})}{1+r}=\lambda q\frac{1-\eta(q)}{\eta(q)}\frac{k}{qf(q)}$$

Combining the first order conditions and substituting $V(\{(\omega,2),2\})$ gives

$$\frac{y_2}{r+\delta}-\frac{r}{r+\delta}\frac{V(\{(b,0),2\})}{1+r}=\frac{k}{qf(q)\eta(q)}$$

Denote the solution to the problem (ω_2^2,q_2^2). The above equations can be expressed as

$$\frac{V(\{(b,0),2\})}{1+r}=\frac{1}{r}\left[y_2-\frac{k(r+\delta)}{q_2^2f(q_2^2)\eta(q_2^2)}\right] \tag{21}$$

Substitute $V(\{(b,0),2\})$ back to V_2^2 gives

$$V_2^2 = \frac{1-\eta(q_2^2)}{\eta(q_2^2)}\frac{k}{q_2^2} + \frac{1}{r}\left[y_2 - \frac{k(r+\delta)}{q_2^2 f(q_2^2)\eta(q_2^2)}\right] \tag{22}$$

The equilibrium cut-off $\bar{c}$ follows:

$$\bar{c} = V_2^2 - V_2^1 = f(q_2^2)\left[\frac{V(\{(\omega,2),2\})}{1+r} - \frac{V(\{(b,0),2\})}{1+r}\right] - f(q_2^1)\left[\frac{V(\{(W(q'),1),2\})}{1+r} - \frac{V(\{(b,0),2\})}{1+r}\right]$$

Substitute the first order conditions from problem $(\mathbf{P-2'})$ and $(\mathbf{P-3})$

$$\bar{c} = \frac{1-\eta(q_2^2)}{\eta(q_2^2)}\frac{k}{q_2^2} - \lambda q_2^1\left[\frac{1-\eta(q_2^1)}{\eta(q_2^1)}\right]\frac{k}{q_2^1} \tag{23}$$

The value of unemployment to a college graduate is given by:c

$$V(\{(b,0),2\}) = b + [1-F(\bar{c})]V_2^1 + F(\bar{c})[V_2^2 - \mathbb{E}(c|c<\bar{c})]$$

Denote $\nu(\bar{c}) \equiv F(\bar{c})[\bar{c} - \mathbb{E}(c|c<\bar{c})]$. Given that $F(\cdot)$ follows exponential distribution, $\nu(\bar{c}) = \bar{c} - F(\bar{c})/\theta$. Rewrite $V(\{(b,0),2\})$ as

$$V(\{(b,0),2\}) - b = V_2^2 - \bar{c} + \nu(\bar{c}) = V_2^1 + \nu(\bar{c})$$

substituting V_2^2 and $V(\{(b,0),2\})$ from equation (22) and equation (21) into the above equation gives

$$\frac{y_2-b}{r+\delta} + \frac{\bar{c}-\nu(\bar{c})}{r+\delta} = \frac{k}{q_2^2 f(q_2^2)\eta(q_2^2)} + \frac{1-\eta(q_2^2)}{\eta(q_2^2)}\frac{k}{(r+\delta)q_2^2} \tag{$Eq-1$}$$

Similarly, substitute V_2^1 and $V(\{(b,0),2\})$ from equation (20) and equation (19) gives

$$\frac{y_2-b}{r+\delta} - \frac{\nu(\bar{c})}{r+\delta} - \frac{k}{q^e f(q^e)\eta(q^e)} = \lambda q_2^1\left(\frac{1-\eta(q_2^1)}{\eta(q_2^1)}\frac{k}{q_2^1}\right)\left[\frac{1}{r+\delta} + \frac{1}{f(q_2^1)}\right] \tag{$Eq-2$}$$

Finally, the zero-profit condition for routine-job employers hiring college graduates in problem $(\mathbf{P-2'})$, stated as a function of q', is given by

$$\frac{k}{q_2^1 f(q_2^1)} = \frac{r+\delta}{[r+\delta+(1-\delta)\lambda_e f(q^e)]}\left[\frac{k}{q^e f(q^e)} - \frac{y_2-y_1}{r+\delta}\right] + \frac{k}{q^e f(q^e)}\left(\frac{1-\eta(q^e)}{\eta(q^e)}\right) \tag{$Eq-3$}$$

The solution to the problems $(\mathbf{P-1})$, $(\mathbf{P-2'})$ and $(\mathbf{P-3})$ are a set of $\{q_2^1, q_2^2, q^e, \omega^e, \omega_2^1, \omega_2^2\}$ such that: $\{q_2^1, q_2^2, q^e,\}$ must solve the above three equations $(Eq-1)$ to $(Eq-3)$, with $\bar{c}$ given by equation (23); $\{\omega^e, \omega_2^1, \omega_2^2\}$ are characterized by the following equations:

$$\omega^e = y_2 - \frac{k(r+\delta)}{q^e f(q^e)} \tag{24}$$

$$\omega_2^2 = y_2 - \frac{k(r+\delta)}{q^e f(q^e)} \tag{25}$$

$$\omega_2^1 = y_1 - \frac{k\left[r+\delta+(1-\delta)\lambda_e f(q^e)\right]}{q_2^1 f(q_2^1)} \tag{26}$$

I now show that a solution to the problems exists, using the following steps. First, I show that for a given $\bar{c}$, a solution exists to equations $(Eq-1)$ to $(Eq-3)$. Next, I construct $D(\bar{c}) = V_2^2 - V_2^1$ using the values of $\{q_2^1, q_2^2, q^e\}$ implied with a fixed $\bar{c}$ and show that a fixed point $D(c_0) = c_0$ exists, with $c_0 > 0$.

Lemma 3 *For any given $\bar{c} \in [0, c_b]$, a solution to equation $(Eq-1)$ to $(Eq-3)$ exists with $q^e(\bar{c}) \in (q_c(\bar{c}), q_d)$, where $q_c(\bar{c})$ solves the following equation of q_c*

$$\frac{y_2 - b}{r+\delta} - \frac{\nu(\bar{c})}{r+\delta} = \frac{k}{q_c f(q_c)\eta(q_c)}$$

and q_d is such that

$$\frac{r+\delta}{r+\delta+(1-\delta)\lambda_e f(q_d)}\left[\frac{k}{q_d f(q_d)} - \frac{y_2-y_1}{r+\delta}\right] + \frac{k}{q_d f(q_d)}\left(\frac{1-\eta(q_d)}{\eta(q_d)}\right) = k$$

and c_b is such that $q_c(c_b) = q_d$.

Proof: Consider first the equation $(Eq-1)$. The right-hand side of the equation is a decreasing function of q_2^2. If q_2^2 converges to 0, the right-hand side converges to ∞ and if q_2^2 converges to ∞, it converges to $k \leq (y_2 - b)/(r+\delta)$. Since $\bar{c} - \nu(\bar{c}) \geq 0$ for $\bar{c} \geq 0$ and $\lim_{\bar{c}\to\infty}(\bar{c} - \nu(\bar{c})) = 1/\theta < \infty$, for any $\bar{c} \geq 0$ there exists a unique $q_2^2 < \infty$ such that equation $(Eq-1)$ holds.

Now consider equation $(Eq-2)$. First, differentiate λq_2^1 and notice that $\partial\lambda q_2^1/\partial q^e < 0$ if and only if $\hat{V}''(q)\hat{S}'(q) > \hat{V}'(q)\hat{S}''(q)$. This condition holds since $\hat{S}(q) - \hat{V}(q)$ is decreasing and convex, therefore $\hat{S}'(q) < \hat{V}'(q)$ and $\hat{S}''(q) > \hat{V}''(q)$.

Given that $\partial\lambda q_2^1/\partial q^e < 0$, equation $(Eq-2)$ characterizes q_2^1 as a declining function of q^e. If q_2^1 converges to 0, the right-hand side of the equation converges to infinity and if q_2^1 converges to infinity, it converges to 0. Let $q_c(\bar{c})$ be the unique level of q^e, so that the left-hand side of the equation is equal to zero for any given $\bar{c}$, i.e.

$$\frac{y_2 - b}{r+\delta} - \frac{\nu(\bar{c})}{r+\delta} = \frac{k}{q_c f(q_c)\eta(q_c)}$$

The right-hand side of the above equation is a decreasing function of q_c. As q_c converges to zero, the right-hand side of the equation converges to infinity and when q_c converges to infinity, it converges to k. Define c_a such that

$$\frac{y_2 - b}{r+\delta} - \frac{\nu(c_a)}{r+\delta} = k$$

For any $\bar{c} \in [0, c_a)$, there exists a $q_c(\bar{c}) < \infty$ such that for any $q^e(\bar{c}) > q_c(\bar{c})$, there is a $q_2^1 < \infty$ solves equation $(Eq-2)$.

Finally, consider equation $(Eq-3)$. The left-hand side of the equation increases as q_2^1 increases and the right-hand side of the equation decreases as q^e increases. Therefore, equation $(Eq-3)$ characterize q_2^1 an increasing function of q^e. When q_2^1 converges to 0, the left-hand side of the equation converges to infinity, and when q_2^1 converges to infinity, it converges to k. Define q_d such that

$$\frac{r+\delta}{r+\delta+(1-\delta)\lambda_e f(q_d)}\left[\frac{k}{q_d f(q_d)} - \frac{y_2-y_1}{r+\delta}\right] + \frac{k}{q_d f(q_d)}\left(\frac{1-\eta(q_d)}{\eta(q_d)}\right) = k \tag{27}$$

and $q_2^1 < \infty$ if and only if $q^e < q_d$. It is easy to see that $q_c < q_b$ for any $k > 0$ by comparing equation

(27) and the equation in lemma 1 that characterize q_b.

Given the monotonic property between q_2^1 and q^e, characterized by equation $(Eq-2)$ and $(Eq-3)$, a unique solution exists if and only if $q_c(\bar{c}) < q_d$. Define c_b such that $q_c(c_b) = q_d$. Notice that $c_a > c_b$ since the left-hand side of equation (27) converges to $k - (y_2 - y_2)/(r+\delta) < 0$ as q_d converges to infinity. Therefore, $q_c(c_b) = q_d < q_c(c_a)$ and $c_b < c_a$. For any $\bar{c} \in [0, c_b)$, a unique solution exists to the problem and $0 < q_2^1 < \infty$, $q^e \in (q_b(\bar{c}), q_c)$ and $0 < q_2^2 < \infty$. *QED*

Construct $D(\bar{c}) = V_2^2 - V_2^1$, using the value of V_2^1 and V_2^1 implied by the q_2^2, q_2^1 and q^e as a function of $\bar{c}$. From equation (23), $D(\bar{c})$ can be expressed as

$$D(\bar{c}) = \frac{1-\eta(q_2^2(\bar{c}))}{\eta(q_2^2(\bar{c}))}\frac{k}{q_2^2(\bar{c})} - \lambda q_2^1 \left[\frac{1-\eta(q_2^1(\bar{c}))}{\eta(q_2^1(\bar{c}))}\right]\frac{k}{q_2^1(\bar{c})} \tag{28}$$

To show that fixed point exists for $\bar{c} \in [0, c_b)$, I first show that $D(\bar{c})$ is an increasing function of $\bar{c}$. I then show that $D(c_b) < c_b$ while $D(0) > 0$. Together, a fixed point $D(\bar{c}) = \bar{c}$ exists for $\bar{c} \in [0, c_b)$.

Consider first the monotonicity of $D(\bar{c})$. From equation $(Eq-1)$, q_2^2 increases as $\bar{c}$ increases. The first item in $D(\bar{c})$ is an increasing function of $\bar{c}$. From equation $(Eq-2)$, for any given q^e, an increase of $\bar{c}$ increases q_2^1. Combining with equation $(Eq-3)$, q_2^1 and q^e increases as $\bar{c}$ increases. Given that λq_2^1 is a decreasing function of q^e, the second term after the minus sign decreases as $\bar{c}$ increases. Together, $D(\bar{c})$ is an increasing function of $\bar{c}$.

Now consider the upper bound at $\bar{c} = c_b$. When $\bar{c} = c_b$, $q_c(c_b) = q_d$ and $q_2^1(c_b) = \infty$. Equation $(Eq-2)$ becomes:

$$\frac{y_2 - b}{r+\delta} - \frac{\nu(c_b)}{r+\delta} = \frac{k}{q_d f(q_d)\eta(q_d)} \tag{29}$$

where q_d is defined by equation (27). Substitute the above equation into equation $(Eq-1)$

$$D(c_b) - c_b = \frac{k(r+\delta)}{q_d f(q_d)\eta(q_d)} - \frac{k(r+\delta)}{q_2^2(\bar{c}) f(q_2^2(\bar{c}))\eta(q_2^2(\bar{c}))}$$

$D(c_b) - c_b < 0$ if and only if $q_2^2(\bar{c}) < q_d$. Constructing the following using equation $(Eq-1)$ and (29)

$$\frac{k(r+\delta)}{q_2^2 f(q_2^2(\bar{c}))\eta(q_2^2(\bar{c}))} + \frac{1-\eta(q_2^2(\bar{c}))}{\eta(q_2^2(\bar{c}))}\frac{k}{q_2^2(\bar{c})} - \frac{k(r+\delta)}{q_d f(q_d)\eta(q_d)} - \frac{1-\eta(q_d)}{\eta(q_d)}\frac{k}{q_d} = c_b - \frac{1-\eta(q_d)}{\eta(q_d)}\frac{k}{q_d}$$

Given that $\dfrac{k}{qf(q)\eta(q)} + \dfrac{1-\eta(q)}{\eta(q)}\dfrac{k}{(r+\delta)q}$ is a decreasing function of q, $q_2^2(\bar{c}) < q_d$ would imply that

$$c_b > \left(\frac{1}{\eta(q_d)} - 1\right)\frac{k}{q_d} \tag{30}$$

Lemma 4 *Assume* $(y_1 - b) > \dfrac{\delta}{1+r}(y_2 - y_1)$. *There exists a number* $\bar{k}_1 > 0$ *such that for* $k \in (0, \bar{k}_1]$, $D(c_b) < c_b$.

Proof: Differentiate equation (27). It is easy to verify that $\dfrac{dq_d}{q_k} > 0$. and $\lim_{k\to 0} q_d = 0$. Also notice that

$$\lim_{k\to 0}\frac{k}{q_d f(q_d)} = \lim_{k\to 0}\frac{k}{q_d} = 0$$

and

$$\lim_{k\to 0}\frac{k}{q_d f(q_d)\eta(q_d)} = \frac{y_2 - y_1}{1+r}$$

Since $c_b = \nu(c_b) + F(c_b)/\theta > \nu(c_b)$, a sufficient condition for equation (30) to hold is that

$$\nu(c_b) - \left(\frac{1}{\eta(q_d)} - 1\right)\frac{k}{q_d} = y_2 - b - \frac{k(r+\delta+f(q_c))}{q_d f(q_d)\eta(q_d)} + \frac{k}{q_c} > 0$$

In the limit at $k \to 0$

$$\lim_{k\to 0}\left[\nu(c_b) - \left(\frac{1}{\eta(q_d)} - 1\right)\frac{k}{q_d}\right] = y_1 - b - \frac{\delta}{1+r}(y_2 - y_1)$$

which is positive if $y_1 - b > \dfrac{\delta}{1+r}(y_2 - y_1)$, as assumed. Therefore, by continuity, there is a $\bar{k}_1 > 0$ such that $D(c_b) - c_b < 0$. *QED*

Finally, consider the lower boundary at $\bar{c} = 0$. Denote $q^1 = q_2^1(0)$, $q^2 = q_2^2(0)$ and $q_0^e = q^e(0)$. Rewrite equation $(Eq-1)$ and $(Eq-2)$ as

$$\frac{y_2 - b}{r+\delta} - \frac{r}{r+\delta}\frac{V(\{(b,0),2\})}{1+r} = \frac{k}{q^2 f(q^2)\eta(q^2)} \tag{31}$$

and

$$\frac{y_2 - b}{r+\delta} - \frac{k}{q_0^e f(q_0^e)\eta(q_0^e)} - \frac{r}{r+\delta}\frac{V(\{(b,0),2\})}{1+r} = \lambda q_2^1\left(\frac{1-\eta(q^1)}{\eta(q^1)}\right)\frac{k}{q^1 f(q^1)} \tag{32}$$

First notice that $\lambda q^1 > 1$. From equation (18), $\lambda q^1 > 1$ if and only if $d\hat{S}(q)/dq > 0$, which holds for $q < q_b$, as shown in Lemma 2. Combining equation (31) and (32)

$$\frac{k}{q_0^e f(q_0^e)\eta(q_0^e)} - \frac{k_2}{q^2 f(q^2)} = \left(\frac{1-\eta(q^2)}{\eta(q^2)}\right)\frac{k}{q^2 f(q^2)} - \lambda q^1\left(\frac{1-\eta(q^1)}{\eta(q^1)}\right)\frac{k}{q^1 f(q^1)} \tag{33}$$

substitute equation $(Eq-3)$

$$\begin{aligned}\frac{k}{q^1 f(q^1)} - \frac{k}{q^2 f(q^2)} + \frac{r+\delta}{[r+\delta+(1-\delta)\lambda_e f(q_0^e)]}\left[\frac{y_2 - y_1}{r+\delta} - \frac{k}{q_0^e f(q_0^e)}\right] + \frac{k}{q_0^e f(q_0^e)} \\ = \left(\frac{1-\eta(q^2)}{\eta(q^2)}\right)\frac{k}{q^2 f(q^2)} - \lambda q^1\left(\frac{1-\eta(q^1)}{\eta(q^1)}\right)\frac{k}{q^1 f(q^1)}\end{aligned}$$

Given that $\lambda q_2^1 > 1$ and that the second term is greater than zero, the following inequality must hold

$$\frac{k}{q^1 f(q^1)} - \frac{k}{q^2 f(q^2)} < \left(\frac{1-\eta(q^2)}{\eta(q^2)}\right)\frac{k}{q^2 f(q^2)} - \left(\frac{1-\eta(q^1)}{\eta(q^1)}\right)\frac{k}{q^1 f(q^1)}$$

Since both $\dfrac{k}{qf(q)}$ and $\left(\dfrac{1-\eta(q)}{\eta(q)}\right)\dfrac{k}{qf(q)}$ are decreasing functions of q, it must be that $q^1 > q^2$.

Rewrite equation (33) as

$$\frac{k}{q_0^e f(q_0^e)\eta(q_0^e)} - \frac{k}{q^2 f(q^2)} = \frac{1}{f(q^2)}\left[\left(\frac{1-\eta(q^2)}{\eta(q^2)}\right)\frac{k}{q^2} - \lambda q^1\left(\frac{1-\eta(q^1)}{\eta(q^1)}\right)\frac{k}{q^1}\frac{f(q^2)}{f(q^1)}\right] = \frac{1}{f(q^2)}D(0)$$

and $D(0) > 0$ if $\dfrac{k}{q_0^e f(q_0^e)\eta(q_0^e)} > \dfrac{k}{q^2 f(q^2)}$.

Using equation $(Eq-1)$, was evaluated at $\bar{c}=0$. Since the right-hand side of the equation decreases with q^2, $dq^2/dk > 0$, it is easy to show that $\lim_{k\to 0} q_2^2 = \lim_{k\to 0} k/q^2 f(q^2) = 0$.

As showed in the last lemma that $\lim_{k\to 0}\dfrac{k}{q_c f(q_c)\eta(q_c)} = \dfrac{y_2-y_1}{1+r}$. Also given that $q^e(\bar{c})$ is an increasing function of $\bar{c}$,

$$\lim_{k\to 0}\frac{k}{q_0^e f(q_0^e)\eta(q_0^e)} > \lim_{k\to 0}\frac{k}{q_c f(q_c)\eta(q_c)} = \frac{y_2-y_1}{1+r}$$

As a result, there exists an $\bar{k}_2 > 0$ that for any $k \in (0, \bar{k}_2]$, $D(0) > 0$.

Overall, assuming $(y_1-b)/(y_2-y_1) > \delta/(1+r)$, there is a number $\bar{k} > 0$ such that for any $k \in (0, \bar{k}]$, there exists a unique set of $\{q_{12}^*, q_{22}^*, q_e^*, \omega_{12}^*, \omega_{22}^*, \omega_e^*\}$ that solve the problems $(\mathbf{P-1})$ to $(\mathbf{P-3})$. The solution must be one of the two cases: if the solution to equations $(Eq-1)$ to $(Eq-3)$ has $q^e \geq q_a$, an interior solution solves the problems and is characterized by equation $(Eq-1)$ to $(Eq-3)$, together with equations (24) to (26). Otherwise, a corner solves the problem where $q_e^* = q_a$ while q_{12}^* and q_{22}^* are characterized by equations $(Eq-1)$ and $(Eq-2)$ evaluated at $q^e = q_a$, together with equations (24) to (26).

High School Graduates

Consider the optimal search problem $(\mathbf{P-4})$ of a high school graduate. Ignoring the incentive compatibility constraint, the first order conditions for an interior solution are given by

$$\lambda q = 1$$

and

$$\frac{V(\{(\omega,1),1\})}{1+r} - \frac{V(\{(b,0),1\})}{1+r} = \lambda q\frac{1-\eta(q)}{\eta(q)}\frac{y_1-\omega}{r+\delta}$$

Substituting $V(\{(\omega,1),1\})$ and the zero-profit condition gives

$$\frac{y_1}{r+\delta} - \frac{r}{r+\delta}\frac{V(\{(b,0),1\})}{1+r} = \frac{k}{qf(q)\eta(q)} \tag{34}$$

Substitute $V(\{(b,0),1\})$ from the above back to the first order condition, an interior solution to the problem must satisfy

$$y_1 - b = \frac{k}{qf(q)}\left[r+\delta+\left(\frac{1-\eta(q)}{\eta(q)}\right)[r+\delta+f(q)]\right] \tag{35}$$

The right-hand side is a decreasing function of q, it converges to zero when q converges to infinity and it converges to $k(r+\delta)$. There must exist a $q<\infty$, which solves the problem. The wage ω follows

$$\omega = y_1 - \frac{(r+\delta)k}{qf(q)} \tag{36}$$

If the incentive compatibility constraint is binding, then equilibrium allocation of high school graduates is such that the college graduates are indifferent between mimicking the high school graduates versus the optimal allocation $(\omega_{12}^*, q_{12}^*)$ when applying for routine jobs. Let (q_1, ω_1) be the allocation such that the college graduate is indifferent,

$$V_2^1 = f(q_1)\left[\frac{V(\{(\omega_1, 1), 2\})}{1+r} - \frac{V(\{(b,0),2\})}{1+r}\right] + \frac{V(\{(b,0),2\})}{1+r} \tag{37}$$

with the zero-profit condition

$$\omega_1 = y_1 - \frac{(r+\delta)k}{q_1 f(q_1)} \tag{38}$$

Denote $\{q_1^*, \omega_1^*\}$ the solution to problem $(\mathbf{P}-\mathbf{4})$, which must be one of the two cases. If the incentive compatibility constraint is not binding, the solution to problem $(\mathbf{P}-\mathbf{4})$ is interior and is characterized by equations (45) and (43). Otherwise, the solution to the problem is constrained and characterized by equations (37) and (38).

It is now straight forward to characterize the distribution of workers ψ.

$$\psi(\{(b,0),2\}) = \frac{\delta}{\delta + (1-F(c_0))f(q_{12}^*) + F(c_0)f(q_{22}^*)}\phi$$

$$\psi(\{(\omega_{22}^*, 2), 2\}) = \frac{F(c_0)f(q_{22}^*)}{\delta}\psi(\{(b,0),2\})$$

$$\psi(\{(\omega_{12}^*, 1), 2\}) = \frac{(1-F(c_0))f(q_{12}^*)}{\delta + (1-\delta)\lambda_e f(q_e^*)}\psi(\{(b,0),2\})$$

$$\psi(\{(\omega_e^*, 2), 2\}) = \frac{(1-\delta)\lambda_e f(q_e^*)}{\delta}\psi(\{(\omega_{12}^*, 1), 2\})$$

$$\psi(\{(b,0),1\}) = \frac{\delta}{\delta + f(q_1^*)} * (1-\phi)$$

$$\psi(\{(\omega_1^*, 1), 1\}) = (1-\phi) - \psi(\{(b,0),1\})$$

This concludes the proof of **Proposition 2**. **QED**

Proof of Proposition 3

I show a sufficient condition for the incentive compatibility constraint to bind, by constructing an example at the limit. First, take $\theta \to 0$, the average cost of applying for cognitive job goes to infinity. In the limit, all college graduates apply for routine jobs only when unemployed, and search on-the-job once employed.

Let the cost of posting a poaching offer to be k_p, while the cost of posting when hiring unemployed workers remains to be k. Take $k_p \to 0$ such that poaching employed workers is free. The job arrival rate of on-the-job search $f(q^e)$ converges to 1, and the poaching wage ω^e converges to the productivity y_2. Once matched with a routine job, the worker stays only for one period and switches employer and receives y_2 in the next period. Both the quite rate and the poaching wage are independent of the current wage.

The value of employment in a routine job with wage ω for college graduate is:

$$V(\{(\omega,1),2\}) = \omega + (1-\delta)\lambda_e \frac{V(\{(y_2,2),2\})}{1+r} + (1-\delta)(1-\lambda_e)\frac{V(\{(\omega,1),2\})}{1+r} + \delta\frac{V(\{(b,0),2\})}{1+r}$$

substituting $V(\{(y_2,2),2\}$ gives

$$\frac{V(\{(\omega,1),2\})}{1+r} = \frac{\omega}{r+\delta+(1-\delta)\lambda_e} + \frac{(1-\delta)\lambda_e}{r+\delta+(1-\delta)\lambda_e}\frac{y_2}{r+\delta} + \frac{\delta}{r+\delta}\frac{V(\{(b,0),2\})}{1+r}$$

Consider now the optimal search problem of an unemployed college graduate when applying for a routine job

$$\max_{q,\omega} f(q)\frac{V(\{(\omega,1),2\})}{1+r} + [1-f(q)]\frac{V(\{(b,0),2\})}{1+r}$$

s.t.

$$-k + qf(q)\frac{y_1-\omega}{r+\delta+(1-\delta)\lambda_e} = 0$$

Note that the discount rate of firms is $r+\delta+(1-\delta)\lambda_e$ given that the quit rate is $(1-\delta)\lambda_e$. The corresponding first order conditions are:

$$\lambda q = 1$$

where λ is the Lagrange multiplier

$$\frac{V(\{(\omega,1),2\})}{1+r} - \frac{V(\{(b,0),2\})}{1+r} = \lambda q\frac{1-\eta(q)}{\eta(q)}\frac{y_1-\omega}{r+\delta+(1-\delta)\lambda_e}$$

Combining the first order conditions and substituting the zero-profit condition to get

$$\frac{V(\{(\omega,1),2\})}{1+r} - \frac{V(\{(b,0),2\})}{1+r} = \frac{1-\eta(q)}{\eta(q)}\frac{k}{qf(q)}$$

Let (q_2,ω_2) denote the solution to the problem, q_2 and ω_2 satisfy

$$\frac{y_1}{r+\delta+(1-\delta)\lambda_e} + \frac{(1-\delta)\lambda_e}{r+\delta+(1-\delta)\lambda_e}\frac{y_2}{r+\delta} - \frac{r}{r+\delta}\frac{V(\{(b,0),2\})}{1+r} = \frac{k}{q_2 f(q_2)\eta(q_2)} \tag{39}$$

and

$$\omega_2 = y_1 - \frac{k(r+\delta+(1-\delta)\lambda_e)}{q_2 f(q_2)} \tag{40}$$

The value of unemployment for college graduates $V(\{(b,0),2\}$ equals

$$V(\{(b,0),2\}) = b + f(q_2)\left[\frac{V(\{(\omega_2,1),2\})}{1+r} - \frac{V(\{(b,0),2\})}{1+r}\right] + \frac{V(\{(b,0),2\}}{1+r}$$

substituting the first order conditions such that

$$\frac{r}{1+r}V(\{(b,0),2\}) = b + \frac{1-\eta(q_2)}{\eta(q_2)}\frac{k}{q_2}$$

substituting this back to (39), q_2 must solve

$$\frac{y_1}{r+\delta+(1-\delta)\lambda_e}+\frac{(1-\delta)\lambda_e}{r+\delta+(1-\delta)\lambda_e}\frac{y_2}{r+\delta}=\frac{1}{r+\delta}\left(b+\frac{1-\eta(q_2)}{\eta(q_2)}\frac{k}{q_2}\right)+\frac{k}{q_2 f(q_2)\eta(q_2)} \tag{41}$$

Similarly, consider the unconstrained optimal search problem of an unemployed high school graduate

$$\max_{q,\omega} f(q)\frac{V(\{(\omega,1),1\})}{1+r}+[1-f(q)]\frac{V(\{(b,0),1\})}{1+r}$$

s.t.

$$-k+qf(q)\frac{y_1-\omega}{r+\delta}=0$$

The corresponding first order conditions together imply

$$\frac{V(\{(\omega,1),1\})}{1+r}-\frac{V(\{(b,0),1\})}{1+r}=\frac{1-\eta(q)}{\eta(q)}\frac{k}{qf(q)}$$

Let (q_1,ω_1) denote the solution to the above problem, q_1 and ω_1 satisfy

$$\frac{y_1}{r+\delta}-\frac{r}{r+\delta}\frac{V(\{(b,0),1\})}{1+r}=\frac{k}{q_1 f(q_1)\eta(q_1)} \tag{42}$$

and

$$\omega_1=y_1-\frac{k(r+\delta)}{q_1 f(q_1)} \tag{43}$$

The return to search for an unemployed high school graduate U_1 follows

$$V(\{(b,0),1\})=b+f(q_1)\left[\frac{(V(\{(\omega_1,1),1\}))}{1+r}-\frac{V(\{(b,0),1\})}{1+r}\right]+\frac{U_1}{1+r}$$

substitute the FOC, one obtains

$$\frac{r}{1+r}V(\{(b,0),1\})=b+\frac{1-\eta(q_1)}{\eta(q_1)}\frac{k}{q_1} \tag{44}$$

and q_1 solves

$$\frac{y_1}{r+\delta}=\frac{1}{r+\delta}\left(b+\frac{1-\eta(q_1)}{\eta(q_1)}\frac{k}{q_1}\right)+\frac{k}{q_1 f(q_1)\eta(q_1)} \tag{45}$$

Compare equation (41) and (45), when $y_2=y_1$, the left-hand sides of the two equations are equals, which implies $q_1=q_2$. Also compare equations (40) and (43), when $q_1=q_2$, $\omega_1>\omega_2$. Since $V(\{(\omega,1),2\})$ increases with ω. college graduates would be strictly better off applying to $\{\omega_1,q_1\}$ and search on-the-job, which means

$$f(q_2)\left[\frac{V(\{(\omega_2,1),2\})}{1+r}-\frac{V(\{(b,0),2\})}{1+r}\right]<f(q_1)\left[\frac{V(\{(\omega_1,1),2\})}{1+r}-\frac{V(\{(b,0),2\})}{1+r}\right]$$

Notice that when $q_1=q_2$, $V(\{(b,0),1\})=V(\{(b,0),2\})$ and $V(\{(\omega_2,1),2\})=V(\{(\omega_1,1),1\})$. When $y_1=y_2$, the total surplus of a match, the workers' share of surplus are equal for both types of workers, even though the wages received out of unemployment are not the same.

So far, I have shown that, at the limit when $\theta=0$, $k_p=0$ and $y_2=y_1$, college graduates have strict incentive to mimic high school graduates. Fixing y_1, since the problem is continuous in y_2 (see equation

(41)), there exists a level $\bar{y} > y_1$ such that for all $y_2 \in (y_1, \bar{y})$, the incentive compatibility constraint binds.

Next I show that the problem is also continuous in θ. Since θ enters the problem of college graduates through the cut-off cost $\bar{c}$ with $\nu(\bar{c}) = \bar{c} - F(\bar{c})/\theta$, the problem is continuous in θ. Therefore, there exist a $\bar{\theta}$, such that at $k_p = 0$, the incentive compatibility constraint binds when $y_2 \in (y_1, \bar{y})$ and $\theta \in (0, \bar{\theta})$. Finally, I show that the problem is also continuous in k_p. Rewrite the condition in Lemma one in k_p:

$$q^e f(q^e) \left(\frac{y_2 - \omega^e}{r + \delta} \right) = k_p$$

$$\frac{\omega^e - \omega_2}{r + \delta + (1 - \delta)\lambda_e f(q^e)} = \frac{1 - \eta(q)}{\eta(q)} \frac{k_p}{q^e f(q^e)}$$

where ω_2 is the wage of a college graduate employed in routine jobs and ω^e is her wage the after on-the-job and q^e if associated queue. It is clear that both q^e and ω^e are continuous in k_p. As $k_p \to 0$, $q^e \to 0$. Since $\omega^e - \omega_2$ is greater than 0, therefore $k_p/q^e f(q^e) \to 0$ and $w^e \to y_2$.

There exist numbers $\bar{k}_1 > 0$, $\bar{\theta} > 0$ and $\bar{y} > y_1$ such that for all $k \in (0, \bar{k}_1)$, $\theta \in (0, \bar{\lambda})$ and $y_2 \in (y_1, \bar{y})$, the incentive compatibility constraint is binding.

Now I show that in the limit case when the incentive compatibility constraint is binding, the equilibrium unemployment rate of high school graduates is inefficiently high. Denote (ω_1^*, q_1^*) the equilibrium allocation of high school graduates. The allocation is such that college graduates are indifferent between applying to (ω_2, q_2) and (ω_1^*, q_1^*), i.e.

$$f(q_2) \left[\frac{V(\{(\omega_2, 1), 2\})}{1 + r} - \frac{V(\{(b, 0), 2\})}{1 + r} \right] = f(q_1^*) \left[\frac{V(\{(\omega_1^*, 1), 2\})}{1 + r} - \frac{V(\{(b, 0), 2\})}{1 + r} \right] \tag{46}$$

The allocation also generates zero expected profits to routine employers anticipating high school graduates, such that

$$\omega_1^* = y_1 - \frac{k(r + \delta)}{q_1^* f(q_1^*)}$$

The above two equations have two potential solutions. One has a larger queue than q_1 while the other has a smaller queue. To see which possible solution is the equilibrium allocation, compare the slopes of indifference curves between high school and college graduates. The indifference curves have positive slopes in the wage-queue space, because workers prefer both a higher wage and a lower queue. If the indifferent curve of college graduates has a steeper slope in $d\omega/dq$, then high school graduates would strictly prefer the allocation with a higher queue.

Consider the slope of indifference curve of high school graduates, evaluated at (ω_1^*, q_1^*), which is

$$\left(\frac{d\omega}{dq} \right)^1 \Bigg|_{(\omega_1^*, q_1^*)} = -f'(q_1^*)(r + \delta) \left[\frac{V(\{(\omega_1^*, 1), 1\})}{1 + r} - \frac{V(\{((b, 0), 1)\})}{1 + r} \right]$$

The slope of indifference curve of college graduates, evaluated at (ω_1^*, q_1^*), is listed below. Notice that in the limit case, an on-the-job decision does not depend on the current wage

$$\left(\frac{d\omega}{dq}\right)^2\Bigg|_{(\omega_1^*,q_1^*)} = -f'(q_1^*)(r+\delta)\left[\frac{V(\{(\omega_1^*,1),2\})}{1+r} - \frac{V(\{((b,0),2)\})}{1+r}\right]$$

In the limit when $\theta = 0$, $k_p = 0$ and $y_2 = y_1$, $V(\{(b,0),2\}) = V(\{(b,0),1\})$, while $V(\{(\omega_1^*,1),2\}) > V(\{(\omega_1^*,1),1\})$, as discussed earlier. The slope of indifference curve of a college graduate is higher than that of a high school graduate. This means compared with college graduates, high school graduates would prefer a larger increase in queue for a marginal increase in wage. For the two allocations that college graduates are indifferent (both satisfy equation (46)), high school graduates would strictly prefer the one with a large queue. Therefore, $q_1^* > q_1$.

Since the unemployment rate of high school graduates equals to $u_1^* = \delta/[\delta + f(q_1^*)]$, $u_1^* > u_1 = \delta/[\delta+f(q_1)]$. The equilibrium unemployment rate of high school graduate is higher than the constrained efficient level.

As showed earlier, the problem is continuous in θ, k_p and y_2, therefore there exists some cut-offs $\bar{k}_1 > 0$, $\bar{\theta} > 0$ and $\bar{y}$ such that the incentive compatibility constraint is binding and the unemployment rate of high school graduates is inefficiently high. *QED*

Proof of Proposition 4

Consider a pooling equilibrium with (q_p, μ_p, ω_p) in the routine job market. Since firms earn zero expected profits in equilibrium, they must cross-subsidize and make positive profits with high school graduates, such that

$$k < P_1(q_p, \mu_p)\frac{y_1 - \omega_p}{r+\delta} \tag{47}$$

Consider a hypothetical deviation with (q', μ', ω') in the routine job market, where $\omega' = \omega_p + \epsilon$ and μ' is such that there are only high school graduates applying to the deviation market. Let q_i' be the queue length such that a type i worker is indifferent between the equilibrium market and the deviating market. As showed in Proposition 1, q_1' must be larger than q_2', since a marginal increase of queue length affects the matching rate for both workers equally, while a marginal increase of wage has a larger effect on increasing the return to search for high school graduates. For any $q' \in [q_2', q_1')$, the market (q', ω') attracts only high school graduates.

Next, I show that there exist a pair of (q', ω') such that firms earn a positive profit while only high school graduates are attracted. Notice that $P_1(q_p, \mu_p) < P_1(q_p, \mu')$ and firms earn positive profits with (q_p, μ', ω_p). As ϵ converges to zero, q_1' converges to q_p, and firms earn positive profits (q_1', μ', ω'). Therefore there always exist an $\epsilon > 0$ such that the set (q', μ', ω') generate profit profits for firms with routine jobs while attracting only high school graduates to apply with $q' < q_1'$. This implies that there is always a profitable deviation violating the equilibrium refinement and the pooling set (q_p, μ_p, ω_p) cannot be part of an equilibrium. *QED*

Proof of Proposition 5

Consider a pooling equilibrium with (q_p, μ_p, ω_p) in the routine job market. Firms earn zero expected profits in equilibrium, and must cross-subsidize and earn positive profits with high school graduates under (q_p, μ_p, ω_p), such that

$$k < P_1(q_p, \mu_{1p}) \frac{y_1 - \omega_p}{r + \delta}$$

Also consider a hypothetical deviation where the routine job market is (q', μ', ω'), such that $q' = q_p$, $\mu'_1 = 1$. Let ω'_1 be the wage such that a high school graduate is indifferent between the deviation allocation and the equilibrium allocation, such that

$$Q_1(q_p, \mu_{1p}) \left[\frac{V(\{(\omega_p, 1), 1\})}{1 + r} - \frac{V(\{(b, 0), 1\})}{1 + r} \right] = Q_1(q', \mu'_1) \left[\frac{V(\{(\omega'_1, 1), 1\})}{1 + r} - \frac{V(\{(b, 0), 1\})}{1 + r} \right]$$

where $Q_1(q, \mu_1)$ equals $1/(1 + \sigma\mu_1 q + (1 - \sigma)q)$. The above equation can be rewritten as

$$\frac{\omega'_1 - \omega_p}{r + \delta} = \frac{\sigma q_p (1 - \mu_{1p})}{1 + \sigma\mu_{1p} q_p + (1 - \sigma) q_p} \left[\frac{V(\{(\omega_p, 1), 1\})}{1 + r} - \frac{V(\{(b, 0), 1\})}{1 + r} \right] \tag{48}$$

Similarly, for college graduates, let ω'_2 be the wage such that college graduates are indifferent between applying to the deviation market and the equilibrium market. Notice that the deviation differs from the equilibrium only in the routine job market. Let $V'(\{(\omega', 1), 2\})$ be the value function when a college graduate searches in the deviating market for routine jobs, and continues to search on the job for the same ω^e_p and q^e_p as in the equilibrium. The wage ω'_2 must satisfy

$$Q_2(q_p, \mu_{1p}) \left[\frac{V(\{(\omega_p, 1), 2\})}{1 + r} - \frac{V(\{(b, 0), 2\})}{1 + r} \right] = Q_2(q', \mu'_1) \left[\frac{V'(\{(\omega', 1), 2\})}{1 + r} - \frac{V(\{(b, 0), 2\})}{1 + r} \right]$$

where $Q_2(q, \mu_1)$ equals $\dfrac{1 + q - \sigma q}{(1 + q)\left[1 + \sigma\mu_1 q + (1 - \sigma)q\right]}$. Substituting Q_2, the above equation can be simplified as

$$\frac{\omega'_2 - \omega_p}{r + \delta + (1 - \delta)\lambda^e f(q^e_p)} = \frac{\sigma q_p (1 - \mu_{1p})}{1 + \sigma\mu_{1p} q_p + (1 - \sigma) q_p} \left[\frac{V(\{(\omega_p, 1), 2\})}{1 + r} - \frac{V(\{(b, 0), 2\})}{1 + r} \right] \tag{49}$$

Comparing equation (48) and (49), it is clear that $\omega'_2 > \omega'_1$ due to both $\lambda^e f(q^e_p) > 0$ and $V(\{(\omega_p, 1), 2\}) - V(\{(b, 0), 2\}) > V(\{(\omega_p, 1), 1\}) - V(\{(b, 0), 1\})$. The second inequality compares the workers' surplus when matched with a routine job. It holds because the joint surplus for matching is higher for college graduates due to the continuation value from job-to-job transition into more productive cognitive jobs, while the firm's surplus are the constant for both types of workers. Therefore, the deviation attracts only high school graduates if $\omega' \in (\omega'_1, \omega'_2)$.

Let $\bar{\omega}$ be the wage such that firms earn zero profit with routine jobs in market $(q', \mu', \bar{\omega})$, then $\bar{\omega}$ must satisfy

$$k = P_1(q', \mu'_1) \frac{y_1 - \bar{\omega}}{r + \delta}$$

According to the equilibrium refinement, if the deviation market (q', μ', ω') can generate positive profit for firms $(\omega' < \bar{\omega})$ and attracts only high school workers $(\omega'_1 < \omega' < \omega'_2)$, then the pooling set

(q_p, μ_p, ω_p) cannot be part of an equilibrium. Such ω' exists if $\bar{\omega} > \omega'_1$, and I show that it is the case when σ converges to zero. Since $P_1(q_p, \mu_{1p}) = \frac{\mu_{1p} q_p}{1+q_p}$ when $\sigma = 0$ and, $P_1(q', \mu'_1) = \frac{q_p}{1+q_p}$, $\bar{\omega} > \omega_p$ as σ converges to zero. Also, because ω'_1 decreases in σ and converges to ω_p as σ converges to zero (equation (48)), there always exists an $\sigma > 0$ such that $\omega' \in (\omega'_1, \bar{\omega})$, and (q_p, μ_p, ω_p) cannot be an equilibrium market. *QED*

Proof of Proposition 6

Given that the economy suffers from adverse selection, the return to search for unemployed workers with vocational education is greater than the return to search for high school graduates, $U_v - U_h > 0$. From equations (14) and (15), the cutoffs of post-secondary education are $\bar{e} = (U_v - U_h)/\beta$ and $\underline{e} = (U_c - U_v)/(\alpha - \beta)$.

Denote $\bar{\beta}$ such that $\bar{e} = \underline{e}$. When the cost rate of vocational equals to $\bar{\beta}$, an individual with a cost of education greater than $\bar{e}$ does not pursuit post-secondary education. An individual with a cost less or equal to $\bar{e}$ weakly prefers college education. $\bar{\beta}$ solves:

$$\bar{\beta} = \alpha \frac{U_v - U_h}{U_c - U_h} \tag{50}$$

Given that $U_v > U_h$ and $U_c > U_h$, $\bar{\beta} > 0$ for any $\alpha > 0$. The demand for vocational education is positive when $\beta < \bar{\beta}$. *QED*

Proof of Proposition 7

Assume that the economy suffers from adverse selection. With a fixed α, consider the cutoffs as function of β. Let $\bar{e}(\beta) = (U_v - U_h)/\beta$. $\bar{e}(\beta)$ decreases with β such that $\bar{e}'(\beta) < 0$. Similarly, let $\underline{e}(\beta) = (U_c - U_v)/(\alpha - \beta)$ and $\underline{e}(\beta)$ increases with β such that $\bar{e}'(\beta) > 0$. Also consider the ex-ante utility of a worker U_0 as the function of β such that

$$U_0(\beta) = G(\underline{e}(\beta))U_c + [G(\bar{e}(\beta)) - G(\underline{e}(\beta))]U_v + [1 - G(\bar{e}(\beta))]U_h - \beta \mathbb{E}[e|\underline{e}(\beta) \le e < \bar{e}(\beta)] - \alpha \mathbb{E}[e|e \le \underline{e}(\beta)]$$

Differentiate $U_0(\beta)$ with respect to β

$$\frac{dU_0(\beta)}{d\beta} = -\int_{\underline{e}(\beta)}^{\bar{e}(\beta)} eg(e)de \tag{51}$$

where $g(\cdot)$ is the density function of G.

I now consider the ex-ante utility in an economy where the cost of vocational education is $\bar{\beta}$, where $\bar{\beta}$ is defined in Proposition 6. When $\beta = \bar{\beta}$, no individual takes vocational education because the cost is too high. $U_0(\bar{\beta})$ equals the level of ex-ante utility if workers can choose either high school or college with cost rate α. Notice that $\bar{e}(\bar{\beta}) = \underline{e}(\bar{\beta})$, so $\left.\frac{dU_0(\beta)}{d\beta}\right|_{\beta=\bar{\beta}} = 0$.

For any $\beta < \bar{\beta}$, $\bar{e}(\beta) > \underline{e}(\beta)$, therefore $\frac{dU_0(\beta)}{d\beta} < 0$ for $\beta \in (0, \bar{\beta})$. By continuity, $U_0(\bar{\beta}) < U_0(\beta)$ for any $\beta \in (0, \bar{\beta})$. The ex-ante utility of a worker is higher in an economy with positive demand for vocational education than in the economy where no workers can take vocational education. *QED*

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