The unemployment rate of high school graduates

Yu Chen[*]

October 2017

Abstract

In this paper, I show that inefficiently high unemployment of high-school workers can be understood as the labor-market response to an adverse selection problem. The adverse selection problem arises because employment contracts do not systematically discriminate against education, even though over-qualified workers are relatively more likely to quit routine jobs, whose skill requirement are met by all workers. The labor market equilibrium distorts the labor market outcomes of high school graduates by inefficiently increasing their wage at the expense of higher unemployment rate, in order to separate them from overqualified college graduates. In addition, the labor market response to the adverse selection problem creates a demand for post-secondary vocational education, which is valuable because it acts as an entry barrier that prevents college graduates from using routine jobs as stepping-stones towards better jobs.

[*]Department of Economics, University of Calgary, Social Science building, 2500 University Dr NW, Calgary, AB T2N 1N4. I am indebted to Francisco M. Gonzalez for his support and encouragement. I also thank Philip Curry and Matthew Doyle for their guidance and advice. The paper has benefited from the useful discussion with Lutz-Alexander Busch, Gadi Barlevy, Ana Ferrer, Jean Guillaume Forand, Richard Freeman, Jeffrey Smith and Shouyong Shi.

1 Introduction

There is a common perception that high school graduates are being displaced from jobs involving mainly routine tasks. These jobs are increasingly automated, offshored, or performed by over-qualified workers, that is, workers whose qualifications exceed job requirements. The roles of skill-biased technological change and globalization in the determination of labor market conditions faced by less educated workers are widely recognized,[1] but the role of over-qualification remains unclear. Many studies assume that educated and uneducated workers compete for the same scarce jobs.[2] Yet, firm-level data does not support the view that over-qualified workers directly crowd out less educated workers.[3] Moreover, over-qualified workers are likely to use their current job as a stepping stone towards better jobs. In fact, more than 30 percent of overeducated workers switch into matched jobs within a year.[4] But if over-qualified workers have a relatively higher quit rate, it is unclear why they should be the ones displacing other workers, especially from routine jobs, where differences in productivity are not likely to be very large.

In this paper, I show that displacement of high-school graduates from employment can be understood as the labor-market response to an adverse selection problem. The adverse selection problem arises because employment contracts do not systematically discriminate against education, even though over-qualified workers are relatively more likely to quit those routine jobs. Comparing with high school graduates, overqualified college graduates are less attracted to routine jobs that offer a higher wage but are associated with a lower matching rate. Consequently, the labor market equilibrium generates inefficient unemployment of high school graduates. Moreover, I argue that this mechanism explains why vocational education has a higher market value than it is commonly thought.

I begin by analyzing a frictional labor market where workers with different skills search for jobs with different skill requirements. There are two types of workers: high school and college workers. While both high school and college workers can apply for jobs that involve mainly routine tasks, only college workers can perform jobs with non-routine cognitive tasks. Search is directed rather than random. This means all agents take as given the trade-off, such that a job with a higher wage offer is associated with a lower firm-worker ratio. Once employed, workers can continue to search on-the-job. The dynamic nature of on-the-job search implies that workers with different skills also have distinct career paths. While high school workers can only apply for routine jobs, for college workers, routine jobs have value as a stepping stone toward cognitive jobs. When college workers

[1]See Autor et al. (2003) on routine jobs, Goos et al. (2014) on skill biased technological change versus globalization and Autor et al. (2014) on exposure to international trade.

[2]See Gautier (2002), Albrecht and Vroman (2002), Dolado et al. (2009), Acemoglu and Autor (2011), Beaudry et al. (2013) and Barnichon and Zylberberg (2014).

[3]Gautier et al. (2002) find no evidence at the firm-level that firms upgrade their workforce during low employment years.

[4]See Clark et al. (2014). For an example of stepping stone jobs, consider Van den Berg et al. (2002) where students accept medical assistant positions as stepping-stone jobs while searching for positions to become medical specialists.

apply for routine jobs, they continue to search for better jobs after employed.

Given that college workers have higher quit rate on routine jobs, it is not obvious that employers always prefer college workers. In fact, when productivity gain for higher education is limited, especially for routine-based jobs, hiring an overqualified college worker generates less expected profits than hiring a high school worker. In general, employers could compensate higher turnover with lower wage offers. However, using contracts to exclude overqualified workers is difficult to implement: not only firms need to justify paying more skilled workers with lower wage, it discriminates individual worker based on group behavior of expected quitting. Besides, committing ex-ante not to hire overqualified worker is less credible ex-post. When search and hiring are costly, employers might prefer to accept an overqualified candidate rather than leave the vacant job unfilled. As a result, although education is observable,[5] employment contracts do not perfectly exclude overqualified workers. Adverse selection arises when college workers have incentives to apply for jobs where employers anticipate high school workers, who have lower job turnovers.

I show that any competitive search equilibrium of the model separates high school workers and college workers into different markets when they search for routine jobs. Among routine jobs, college workers are attracted to jobs that are easier to find, have higher turnover and pay lower wage;[6] while high school workers prefer jobs that compensate better, have lower turnover but are also more difficult to get.[7] Workers are separated because when searching from unemployment, they trade-off expected wages and job finding rates differently. Since college workers search for routine jobs as a transition into better jobs, they are willing to accept a relatively lower wage as long as the job-finding rate is sufficiently high. In contrast, high school graduates are more willing to wait longer for a relatively higher wage, given they have longer expected job tenures. Equilibrium sorts workers with different career paths into different markets.

The separation, however, is costly and generates inefficient unemployment for high school workers. To exclude overqualified workers, employers offer distorted contracts to attract high school workers. In fact, equilibrium wage for high school is inefficiently high so that the corresponding job finding rate is inefficiently low, therefore discourages college workers from applying. Holding everything else equal, all workers prefer higher wages. However, college workers are less willing to trade off job finding rate for a higher paying routine job, because they have better on-the-job search options. I show that under certain condition, the incentive compatibility constraint of college workers applying for jobs attracting high school workers is binding. High school workers are

[5] If overqualified workers are concerned about discrimination against more educated workers, they have incentives not to disclose their college credentials.

[6] Wage here represents the total labor income from the job that includes for example benefits and pensions. Jobs with high turnover rate are often less generous in these benefits.

[7] Among low skilled occupations with large employment, the fractions of college graduates are high in occupations such as retail salespersons(24.6%), recreation attendants(23.5%) and bartenders(16.5%), while the share of college graduates in janitors and cleaners(5%), truck drivers(5%) and food preparation(5.4%) are lower. (Vedder et al., 2013)

displaced from employment even though they do not compete directly with college workers for the same job.

In this context, I argue that post-secondary vocational education has a higher market value than is commonly thought. Most studies of the role of education in the labor market focus on high school and college workers. Yet, the increasing role of vocational education, which is the post-secondary education that focuses primarily on providing occupationally specific preparation, is striking. In 2010, for example, about 1,650,000 bachelor degrees and about 1,442,000 vocational education and training credentials were awarded in the U.S. vocational credentials awarded per year increased by 60 percent from 2000 to 2010, growing much faster than the 34 percent increase of bachelor degrees. Besides the obvious benefit of skill development, vocational education acts as an entry barrier because jobs requiring vocational education can exclude over-qualified workers from seeking stepping-stone opportunities. By extending the baseline model to include educational choices, I show that the adverse selection problem creates a demand for vocational education. Since employment contracts cannot discriminate against workers with more education, but can condition on required training, vocational education successfully excludes college workers from seeking stepping-stone opportunities. As a result, employers are able to offer contracts that do not suffer from the distortion of the adverse selection problem. I show that introducing costly vocational education into the labor market makes workers ex-ante better off. Vocational education helps screen workers who are more committed to the occupation and improves labor market efficiency.

Occupational licensing is another example of entry barriers as it is often viewed to have no impact on productivity. According to Kleiner and Krueger (2013), in 2008 nearly 30 percent of the workers with more than high school education, but not a bachelor degree, were required to hold a license. Many popular fields of vocational education such as health care and "trades" in areas such as manufacturing, construction, repair, and transportation, also prepare students to obtain occupational licensing. One concern is that, as an entry barrier, occupational licensing may cause job losses by increasing employment costs. While this concern is widespread in policy papers (see Kleiner (2005)) and in the media, my results imply that occupational licensing also has an important benefit as an entry barrier. Imposing restrictions on entry protects low to middle-skilled workers from the competition of college graduates seeking for stepping-stone jobs. Entry barriers such as vocational education and occupational licensing can mitigate the distortion caused by adverse selection by screening out workers who are not pursuing a career in the particular occupation. This is in sharp contrast to standard competitive market models, where a barrier to entry creates monopoly power and is unambiguously welfare reducing.

Finally, I relate the outcomes of this model to the increase in post-secondary educational attainment over the past few decades. Autor et al. (2010) points out that one puzzle in the U.S. labor market is that the relative supply of college-educated workers is not growing fast enough, given the steep rise in the college-versus-high-school earnings ratio. This paper suggests that much of the increase in post-secondary education may take the form of non-bachelor post-secondary vocational education. The reason for this is that skill-biased technological change has not only increased the return to college, relative to high school, but also the return to vocational education. I simulate the model to examine the effect of a skill-biased technological change that is complementary to cognitive jobs but substitute to routine jobs, combined with an increase in university tuition fees to replicate the education of 2010. I find that while the increasing productivity of cognitive jobs leads to a direct increase in the educational attainment at the university level, it also lowers the return to high school by further displacing high school workers from employment. Therefore such a change also increases the incentive for high school workers to attain both college and vocational education. In addition, the reduction in the productivity of some routine jobs has also worsened the labor market outcome of high school workers, leading to an increased popularity of vocational education. Overall, together with an increase in university tuition, skill-biased technological change can explain the increase of both college degrees and post-secondary vocational education over the past thirty years.

The theory I propose is in contrast with previous work that emphasizes the displacement or "crowding-out" of less-educated workers by high skilled-workers. In the frictionless competitive models of Acemoglu and Autor (2011) and Beaudry et al. (2013), no externality is generated by employing high-skilled workers in less-skilled jobs, since replacing less-educated workers by high-skilled ones is an efficient market adjustment. In models with random matching, such as Gautier (2002), Albrecht and Vroman (2002) and Dolado et al. (2009), search externalities rely crucially on the assumption that high-skilled workers and less-educated workers are matched randomly with the same routine employers. What remains unexplained is whether this displacement exists when workers have incentives to apply to different employers. I show that over-qualified workers create a negative spillover effect on low-skilled workers even when they search for different routine jobs. Allowing workers to direct their search is crucial for understanding the mechanism that underlies the displacement of high school graduates and its policy implications.

The rest of the paper is organized as follows. Section 2 lays out the model. Section 3 characterizes the equilibrium with the displacement effect. Section 4 extends the model with educational choices and discusses the implication on vocational education. I also present a numerical simulation to discuss the implications of skill-biased technological changes. Section 5 concludes.

2 The Model

2.1 Environment

Time is discrete and continues forever. All agents are risk neutral and discount the future at a rate $r > 0$. There is a unit measure of workers, distinguished by their observable education attainment.

Let $i \in \{1, 2\}$ indicate these two types of workers, where type-2 workers can be interpreted as workers with a bachelor degree while type-1 workers represent workers with a high school diploma only. The fraction of college workers is fixed to π. This assumption will be relaxed in an extension in section 4 with endogenous educational choices. There are also two types of jobs and $j \in \{1, 2\}$ indicates a job's type. Type-2 jobs represent jobs that involve mostly non-routine cognitive tasks while type-1 jobs involve mostly routine tasks. Cognitive jobs are more skilled and require workers to have a bachelor degree. The measure of jobs is determined endogenously. A routine job produces y_1. For simplicity, I assume that college and high school workers are equally productive in routine jobs. The main results of the paper still hold if this assumption is relaxed. Only college workers are productivity in a cognitive job, with output $y_2 > y_1$.

Table 1: Match Productivity

	Job	
Worker	Routine	Cognitive
High School	y_1	0
College	y_1	$y_2 > y_1$

Each period is divided into four stages: production, separation, search and matching. At the beginning of a period, all workers are either employed or unemployed. Then employed workers produce and collect wages while unemployed workers receive benefit b. At the separation stage, an exogenous separation shock occurs with probability $\delta > 0$. Once a match is hit by the shock, the worker becomes unemployed and starts job-searching in the next period while the job is destroyed. At the search stage, employed workers and workers who were unemployed at the beginning of the period can search. Meanwhile, firms create job vacancies and post wage contracts for each vacancy. At the matching stage, workers and employers are brought into contacts to form new matches. Following Chen et al. (2017), I assume that when an employed worker receives an outside offer, her current employer can choose to make a counteroffer. Then the worker decides whether to accept the poaching offer or stay with the incumbent employer. Allowing counter offers significantly reduces the dynamics and ensures the tractability of the model.

Since this paper focuses on the labor market outcomes of less educated workers, understanding why college workers search for routine jobs is beyond the scope of discussion presented here.[8] Instead, I assume that when searching for college jobs, an unemployed college worker faces a search cost $c > 0$. The cost reflects the relative difficulty of applying for a cognitive job versus a routine job for a college worker,[9] and is realized before the search stage. The cost c is drawn from an exponential distribution $F(\cdot)$ with parameter θ, and is i.i.d. across workers and over time. In equilibrium, college workers apply for a type-1 job when the cost realization is low and apply for

[8]See Chen et al. (2017) for a model with endogenous mismatch of college worker.

[9]Examples of search costs include networking, cost to obtain a license and traveling, the search cost could apply to both routine and cognitive jobs. I assume that on average applying for cognitive jobs is more costly.

a type-2 job otherwise. I also assume that no cost is incurred when workers search on the job. This assumption is made for simplicity. Even with the presence of some search costs, mismatched workers have incentives to search on-the-job because they can improve their matching quality by applying for better jobs.

Let $s = \{\ell, i\} \in S$ denote the state of a type-i worker with labor force status ℓ, and S be the set of all possible states of a worker. A worker's labor market status specifies if the worker is employed or unemployed, as well as the wage and the job type if the worker is employed. For a worker matched with a type-j job receiving wage ω, her labor market status is a pair $\ell = (\omega, j)$. Let $s_u = \{(b, 0), i\} \in S_u$ denote the state of an unemployed worker. The state of a worker is perfectly observable.

I focus on a labor market where each employer posts a single wage contract conditional on workers' states.[10] The contracting environment is that employers commit to a posted wage, which remains constant until an outside offer is received by the employee. Once an outside offer is received, employers can choose whether to respond and offer a take-it-or-leave-it counter wage. Any renegotiation of the wage is based on mutual agreement.

The critical assumption of the model is that routine-job employers have to offer the same wage to both college and high school workers. In other words, wage offers from routine jobs are not conditional on the type of a worker. While employers have incentives to pay college workers a lower wage for routine jobs, to compensate their high expected turnover, this assumption restricts them from discrimination against workers with more education. It also suggests that routine job employers cannot exclude college workers from applying by offering a zero wage to college workers only. When hiring for routine vacancies, employers could attract applications from both college and high school workers for the same job offer. This particular contractual limitation causes adverse selection: while employers would like to exclude college workers from applying for the same job offers with high school workers, they cannot separate them by contracts. The mechanism works the same if education is not observable.

In practice, paying similarly productive workers with different wages might subject to legal risks, especially when individual workers are penalized by the group behaviors of high expected quitting. Besides, job posts that exclude workers with more education from applying are rare. One reason such exclusion is difficult to implement is that committing not to hire over-qualified workers ex-ante is not credible ex-post. Because hiring takes time and is costly, employers might prefer to hire an overqualified worker once they meet instead of leaving the vacant job unfilled.

[10]In general, firms can post menus that have several contracts conditional on different states of workers.However, the free entry condition implies that in equilibrium, all contracts on a menu must generate zero expected profits. This is equivalent to an equilibrium where each firm posts a single contract to workers of different states in separated markets.

Another reason is that workers can omit credentials that are not related to the job if they concern that employers do not hire overqualified workers. In general, offering discriminating contracts to overqualified workers is difficult to implement in the labor market.

The structure of the labor market is as follows. Each period a continuum of markets may open. A market is characterized by an employment contract x and a market queue length q, which equal the measure of job applicants divided by the measure of vacancies in that particular market. Markets are indexed by contracts. A contract $x = \{(\omega, j), \ell\} \in X$ includes a job offer, which specifies the wage ω and the job type j. It also states the labor market status ℓ of workers that the offer is made to. Notice that contracts are not conditional on the type of a worker. Since high school workers cannot perform cognitive jobs, I assume for simplicity that they do not apply for these jobs. Together with the constraint that routine job employers cannot offer different wages contingent on a worker's type, all offers in the model are made conditional on workers' labor market status only. In other words, while employed and unemployed workers never compete for the same job offer, high school and college workers could search in the same market for routine jobs.

Let $Q : X \to \mathbb{R}^+$ be the queue mapping function, where X is set of all feasible contracts. $Q(x)$ is the queue length associated with market x. Search is directed in the sense that both employers and workers take into account the trade-off between wage and queue length for any given job. Each employer can choose to post any contract $x \in X$, with a flow cost $k > 0$. Workers observe all posted contracts and decide where to search. Workers and firms in the same market are brought into contact by a matching technology. Matching is bilateral so that each worker meets at most one employer and vice versa. Workers who search in a market with market queue length q meet an employer with probability $f(q)$, and employers in the same market meet a worker with probability $qf(q)$. Following the standard search literature, I assume that $f(q)$ is twice differentiable, strictly decreasing and convex, with $f(0) = 1$ and $f(\infty) = 0$. I also assume that $qf(q)$ is strictly increasing and concave, approaching 1 as q converges to ∞. In addition, I assume that the elasticity of the matching function with respect to job creation $\eta(q)$, defined as $-\dfrac{qf'(q)}{f(q)}$ is concave, with $\eta(\infty) = 1$. This assumption is not necessary but simplifies the existence proof of an equilibrium.

I assume that $(r+\delta)k < y_2 - y_1$ to allow the possibility job-to-job transition. On-the-job search is a key labor market feature of overqualified workers. It also plays an important role in the model because without on-the-job search, hiring a college or a high school worker is indifferent to a routine job employer and the adverse selection problem disappears.

2.2 Competitive Search Equilibrium

Let $V(s)$ denote the discounted lifetime income of a worker in state s. A worker receives a flow income and searches for jobs. Let $U(s, c, x, q)$ denote the expected surplus from searching for market with contract x and the queue length q, given the worker's current state s, with c be the realized cost of search, which depends on the type of the job and the type of the worker. Workers direct their search among all markets to maximize the expected future income, taking as given the market queue mapping Q. The value function $V(s)$ of worker is state s satisfies:

$$V(s) = \omega + \frac{\delta V(s_u)}{1+r} + (1-\delta)\left\{\frac{V(s)}{1+r} + \mathbb{E}_c\left\{\max_{x\in X} U(s,c,x,Q(x))|s\right\}\right\} \tag{1}$$

for all s. The flow income $\omega = b$ if the worker is unemployed. For an employed worker, the current match is destroyed with probability δ, in which case the worker becomes unemployed. The surplus from searching for a worker in state $s = \{\ell, i\}$ is given by

$$U(s,c,x,Q(x)) = \begin{cases} (1-i)(1-j)c + f(Q(x))\max\left\{0, \frac{V(s_o)}{1+r} - \frac{V(s)}{1+r}\right\} & \text{if } s \in S_u \\ f(Q(x))\max\left\{0, \frac{V(s_o)}{1+r} - \frac{V(s)}{1+r}, \frac{V(s_c)}{1+r} - \frac{V(s)}{1+r}\right\} & \text{if } s \in S \backslash S_u \end{cases} \tag{2}$$

The search cost c applies only when an unemployed college worker searches for a cognitive job. A worker searches in market $x = \{\ell', \ell\}$ meets with an employer with probability $f(Q(x))$. If she fails to meet with an employer, her state is unchanged. Upon meeting, a worker accepts the offer if $V(s_o) > V(s)$, where $s_o = \{\ell', i\}$ is the state of the worker after accepting the offer.

If the worker is currently employed and searches in market $x = \{\ell', \ell\}$, with $\ell' = (\omega', j')$, the probability that she meets a poaching employer is $f(Q(x))$. Once an outside offer is received, the current employer can respond by offering a counter-wage ω_c. If no counter-offer is made, ω_c equals to zero. If the worker chooses to accept the outside offer, she starts producing with the poaching employer in next period with the state $s_o = \{\ell', i\}$. Otherwise, the worker remains matched with her current employer. Her state becomes $s_c = \{(\omega_c, j), i\}$ if the counter-offer is accepted, where j is the employer type of her current match.

In equilibrium, workers anticipate that $\omega^c = g^r(s, s_o)$, where g_r is firm's optimal retention policy specified below. Let g be the search policy of workers and that $g(s,c)$ is the optimal search decision of a worker in state s with cost realization c. Let $c = 0$ if the worker is employed, g is such that

$$g(s,c) = \underset{x\in X}{\text{argmax}} U(s,c,x,Q(x)) \tag{3}$$

Also denote by g^a the acceptance policy such that $g^a(s, s_o, s_c)$ is the probability that a worker in state s accepts the offer from contract x. Formally, g^a is such that

$$g^a(s, s_o, s_c) = \arg\max_{a\in[0,1]}\{aV(s_o) + (1-a)\max\{V(s), V(s_c)\}\} \tag{4}$$

Next, denote by $H(s)$ the sum of discounted profits from an on-going match to the employer, where $s = \{(\omega, j), i\}$ is the state of the employee. The employer collects the flow profit and anticipates that the worker searches on-the-job if the match is not destroyed. Employers take as given the worker's optimal search policy such that an employed worker in state s search in market $x' = \{g(s,0), s\}$. With probability $f(Q(x'))$, the worker receives an outside offer. The employer then chooses whether to make a counter wage offer ω_c, taking as given the worker's optimal acceptance rule. The acceptance rule specifies that the worker accepts the outside offer with probability $g^a(s, s_o, s_c)$, where s_o and s_c are the states of the worker if the outside offer and the retention offer is accepted, respectively. The present value of the match for the employer $H(s)$ satisfies the following:

$$H(s) = y_j - \omega + (1-\delta)\left\{ f(Q(x')) \max_{\omega_c} \left\{ [1 - g^a(s, s_o, s_c)] \frac{H(s_c)}{1+r} \right\} + [1 - f(Q(x'))] \frac{H(s)}{1+r} \right\} \quad (5)$$

Let $g^r(s, s_o)$ denote a solution to problem (5), the optimal retention policy g^r is contingent on the employee's current state.

Now consider employers with unfilled vacancies. Employers choose how many vacancies to create and what contract to post for each vacancy. Employer pays cost k for creating a vacancy and meets with a worker with probability $Q(x)f(Q(x))$ when posting a contract $x = \{\ell', \ell\}$. Because contracts are conditional on the labor force status but not the type of workers, employers need to form expectations about the distribution of the pool of applicants. Let $\mu(\cdot|x)$ be the conditional distribution function of workers who are attracted by contract x over types. The ex-ante return of posting contract x to an employer $J(x, Q(x), \mu(|x))$ is given by

$$J(x, Q(x), \mu(|x)) = -k + Q(x)f(Q(x)) \sum_{i=1}^{2} \left\{ \mu(s|x) g^a(s, s_o, s_c) \frac{H(s_o)}{1+r} \right\} \quad (6)$$

where $s = \{\ell, i\}$, $s_o = \{\ell', i\}$ and $s_c = \{(g_r(s, s_o), j), i\}$. Firms taking as given the workers' optimal acceptance policy $g^a(s, s_o, s_c)$, which equals the probability that the contract x is accepted by a worker of state s.

Definition 1 *A stationary competitive search equilibrium consists of a set of posted contracts $X^* \subseteq X$, a set of workers' states $S^* \subseteq S$, a market queue mapping $Q : X \to \mathbb{R}^+$, a conditional distribution function $\mu(\cdot|x) : S \to [0,1]$, a distribution $\psi : S \to [0,1]$, such that:*

(i) Workers optimize: V satisfies (1), g satisfies (3) and g^a satisfies (4). $g(s,c) \in X^*$ for all $s \in S^*$ and $S^* = \{s \in S | \psi(s) > 0\}$.

(ii) Firms optimize with free entry: $H(s)$ satisfies (5) and g^r solves (5). Moreover, for all $x \in X$, $J(x, Q(x), \mu(|x)) \leq 0$, with equality if $x \in X^*$.

(iii) Consistent beliefs : $\mu(\cdot|x)$ has support on S^*. For any $x = \{\ell', \ell\} \in X^*$,

$$\mu(\{\ell, i\}|x) = \frac{\psi(\{\ell, i\})}{\sum_{i \in \{1,2\}} \psi(\{\ell, i\})}$$

for any state $\{\ell, i\} \in S$.

(iv) Steady-State conditions are satisfied. For all $s \in S$

$$\int_{S^*} Pr(s_{t+1} = \tilde{s} | s_t = s) d\psi(\tilde{s}) = \int_{S^*} Pr(s_{t+1} = s | s_t = \tilde{s}) d\psi(\tilde{s})$$

where $Pr(s_{t+1}|s_t)$ is the unique distribution associated with g, g^a and g^r.

Condition (i) ensures that workers' search and acceptance policies are optimal for all states, taking as given the market queue length of all contracts and employers' optimal counter-offer strategy. Condition (ii) ensures that employer's retention policies are optimal, taken as given the market queue mapping and the optimal search and acceptance strategy of workers. Free entry implies that equilibrium contracts generate zero expected profits for employers. Condition (iii) ensures that for equilibrium contracts, employers beliefs are consistent with the workers search strategies through Bayes rule on the equilibrium path. Notice that the off-equilibrium belief also has full support on the equilibrium states S^* as in Chen et al. (2017). It implies that an off-equilibrium deviation does not affect the distribution of states in equilibrium. This condition is particularly important given the dynamic nature of the on-the-job search. When a firm posts a deviating poaching contract, it takes as given the allocation of employed workers, even though the pool of workers attracted as well as the associated queue length varies with the deviating contract. Condition (iv) ensures the law of motion for the aggregate state of the economy is stationary. Instead of providing a formal statement of $Pr(s_{t+1} = \tilde{s}|s_t = s)$, I provide the specific context of the equilibrium characterized later in the Appendix.

Equilibrium Refinement

Without imposes restrictions on the belief, the above equilibrium definition allows many equilibria, each supported by a particular belief in markets that are not open and some level of the market queue. For example, an arbitrary equilibrium could be supported when some other markets are not open as no firms post these contracts since they expect to attract no worker, while no worker applies because they anticipate high associated queues. Following closely Chen et al. (2017), I introduce the following refinement to restrict the equilibrium

Definition 2 *A refined equilibrium is a stationary equilibrium such that, for any $x' \notin X^*$, there does not exist a queue length $q' \in \mathbb{R}^+$ and a beliefs $\mu'(\cdot|x')$ on S with support on S^*, such that $J(x', q', \mu'(|x')) > 0$, where for any feasible s, $\mu'(s|x') > 0$ if and only if $U(s, c, x', q') > U(s, c, g(s, c), Q(g(s, c)))$.*

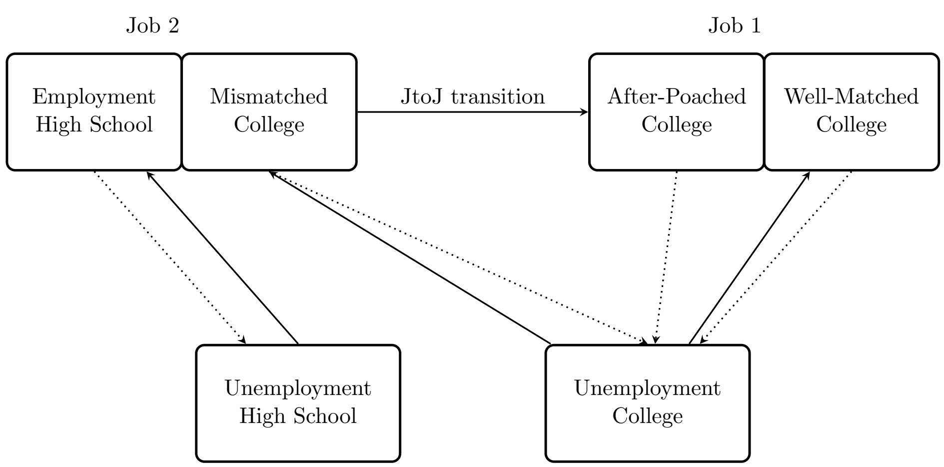
The intuition of the refinement is closely related to the Intuitive Criterion presented by Cho and Kreps (1987). It restricts the beliefs such that positive probabilities cannot be placed on the type of workers whose payoff to the deviation is (weakly) equilibrium-dominated. The refinement eliminates equilibria where some off-equilibrium contracts can generate non-negative profits for the deviating firms while offering the deviating workers above their equilibrium payoff. In other words, for a refined equilibrium, there does not exist a deviating contract that would make some workers better off while keeping all other agents not worse off, for any possible queue and any possible belief as restricted above. This refinement also excludes equilibria where firms do not post certain contracts because they expect no worker to apply while workers do not search for these contracts because they believe the queue length associated with these contracts are too high.

3 Equilibrium with Crowding-out

In this section, I show that a competitive search equilibrium exists, and the equilibrium allocation is unique. I provide a sufficient condition such that adverse selection is present. I discuss the labor market implication of the equilibrium, where college workers displace high school workers out of employment.

Figure 1 depicts the equilibrium dynamics. High school workers search for routine jobs when unemployed. Once matched, they remain employed until hit by the exogenous separation shock. For unemployed college workers, some of them apply for a cognitive job while others search for a routine job, depending on the realization of search cost. College workers matched with routine jobs continue to search on-the-job for a cognitive job. All the college workers matched with cognitive jobs, from unemployment or employment, stay with the employer until the match is destroyed endogenously.

Figure 1: labor Market Flow



The equilibrium dynamics of the labor market is relatively straightforward, given that job-to-job transition occurs only when mismatch college workers search for cognitive jobs. Models with on-the-job search tend to have complicated labor market dynamics with wage ladders (Delacroix and Shi, 2006). In this model, however, the wage ladder does not appear in the equilibrium because employers are allowed to counter outside offers, like in Chen et al. (2017). Under this assumption, incumbent employers have incentives to match any outside offer, up to the worker's matching productivity. As a result, equilibrium poaching wage should be no less than the productivity of a routine job. With non-negative costs of posting vacancies, successful poaching happens only if there is a positive productivity gain from the new match. In equilibrium, poaching offers are made only from cognitive-job employers, hiring college workers currently matched with routine employers. With any negligible small cost of making counteroffers, no retention happens in equilibrium, because the poaching firms can always post a wage that is ϵ higher than what the incumbent firm can compete.

As is standard in the competitive search literature, allocation supported by a competitive search equilibrium can be characterized as the solution to a set of constrained optimization problems.[11] The basic structure of these problems is that workers maximize the expected return to search, subject to firms making non-negative profits. In the presence of adverse selection, the problem is also subject to an incentive compatibility constraint. As shown in the equilibrium dynamics, there are four search problems there. Problem $(\mathbf{P-1})$ presents the on-the-job search decision of an employed worker. The off-job search decision of an unemployed worker looking for a routine and a cognitive job are denoted as $(\mathbf{P-2})$ and $(\mathbf{P-3})$ respectively. Problem $(\mathbf{P-4})$ describes the high school workers' search decision when unemployed. Below I first establish the linkage between these problems and the equilibrium, then I present and discuss the problems in details.

Proposition 1 *i) Any separating equilibrium allocation must solve problems* $(\mathbf{P-1})$ *to* $(\mathbf{P-4})$. *ii) A solution to problems* $(\mathbf{P-1})$ *to* $(\mathbf{P-4})$ *can be supported as an separating equilibrium.*

All formal proofs can be found in the Appendix. Statement $i)$ is straightforward given the definition of the equilibrium refinement. Since the problems are structured such that workers maximize the sum of discounted value given firms making non-negative profits, any allocation that does not solve problems cannot be supported as an equilibrium. The key to show statement $ii)$ is to find a pair of a queue mapping and a belief function that supports the allocation as an equilibrium. Notice that such a pair is not unique. In the Appendix, I provide one example of the pairs. Combining the two statements, Proposition 1 implies that the properties of equilibrium allocation can be understood by studying the set of constrained optimization problems.

I now present these problems. Consider first the on-the-job search problem of a college worker receiving wage ω. Let $V(\{(\omega,1),2\})$ denotes the value function of the workers, such that

[11] See Moen (1997) for a standard competitive search equilibrium, Guerrieri et al. (2010) for a competitive search equilibrium with adverse selection, and Chen et al. (2017) has a competitive search equilibrium with adverse selection and on-the-job search.

$$V(\{(\omega,1),2\}) = \omega + (1-\delta)\max_{\omega',q}\left\{f(q)\frac{V(\{(\omega',2),2\})}{1+r} + [1-f(q)]\frac{V(\{(\omega,1),2\})}{1+r}\right\} + \delta\frac{V(\{(b,0),2\})}{1+r} \quad (\mathbf{P-1})$$

Subject to

$$-k + qf(q)\left(\frac{y_2-\omega'}{r+\delta}\right) \geq 0$$

$$\omega' \geq y_1$$

where the value of a match for a college worker in a cognitive job $V(\{(\omega,2),2\})$ is given by

$$\frac{V(\{(\omega',2),2\})}{1+r} = \frac{\omega'}{r+\delta} + \frac{\delta}{r+\delta}\frac{V(\{(b,0),2\})}{1+r}$$

An employed college worker chooses a pair of wage and queue length (ω', q) to maximize the expected return to search. With probability $f(q)$ the worker meets a new employer and switches to a cognitive job (type 2 job). Otherwise, the worker stays in the current job with the same wage. The value of the new match to the worker is $V(\{(\omega',2),2\})$, which includes the sum of discounted flow incomes and the value of outside option as unemployment. Given the exogenous separation shock, the effective discount rate of a match is $r+\delta$. The first constraint of the problem suggests that employers should make non-negative profits. The total value of a match for the employer is $(y_2-\omega')/(r+\delta)$, where the flow profits are discounted by the effective discount rate. The cost of hiring k should be covered by the expected profits of the firm, which equals the sum of discounted flow profits multiplied by the probability $qf(q)$ of meeting with a worker. The second constraint states that the poaching wage should be no less than the worker's current matching productivity, as discussed earlier.

Denote the solution to the above on-the-job search problem by $\{q^e(\omega), \omega^e(\omega)\}$. Notice that the optimal on-the-job search decision depends on the worker's current wage. The value function of a mismatched college worker with wage ω can then be expressed as:

$$\frac{V(\{(\omega,1),2\})}{1+r} = \frac{\omega}{r+\delta} + \frac{\delta}{r+\delta}\frac{V(\{(b,0),2\})}{1+r} + \frac{(1-\delta)f(q^e(\omega))}{r+\delta}\left[\frac{V(\{(\omega^e(\omega),2),2\})}{1+r} - \frac{V(\{(\omega,1),2\})}{1+r}\right]$$

The total value of a routine job to a college worker has two components: the value of employment and the continuation value from on-the-job search. The first two terms on the left-hand side of the equation present the value of employment, including the sum of discounted flow income and the value of outside option given an exogenous shock. The third term is the expected gain from on-the-job search. With probability $(1-\delta)f(q^e(\omega))$, the worker quit the current employer for a higher wage. The net gain of the job-to-job transition to the worker is $V(\{(\omega^e(\omega),2),2\}) - V(\{(\omega,1),2\})$.

Denote V_2^1 the return to search of an unemployed college worker searching for a routine job, such that

$$V_2^1 = \max_{\omega,q} \left\{ f(q)\frac{V(\{(\omega,1),2\})}{1+r} + [1-f(q)]\frac{V(\{(b,0),2\})}{1+r} \right\} \qquad (\mathbf{P-2})$$

s.t.

$$-k + qf(q)\frac{y_1 - \omega}{r+\delta+(1-\delta)f(q^e(\omega))} \leq 0$$

An unemployed college worker searches for a pair of wage and queue, to maximize the expected return to search. With probability $f(q)$, the worker becomes employed and matches with a routine job in the next period. Otherwise, the worker remains unemployed and searches again. The problem is subject to a non-negative profit constraint. In particular, a filled routine job faces both exogenous and endogenous separation. The endogenous separation arises because college workers continue to search on-the-job, once matched with a routine employer. The flow income of an employers is discount with rate $r+\delta+(1-\delta)f(q^e(\omega))$, where $(1-\delta)f(q^e(\omega))$ is the quit rate of the worker. Notice that the employer takes into consideration that the future quit rate of the worker depends on the level of her current wage.

The last optimization problem of college workers is the problem of unemployed workers applying for a cognitive job. This problem is standard since college workers do not search on-the-job when matched with a cognitive job. Denote V_2^2 the return to search for a cognitive job from unemployment, such that

$$V_2^2 = \max_{(\omega,q)} \left\{ f(q)\frac{V(\{(\omega,2),2\})}{1+r} + [1-f(q)]\frac{V(\{(b,0),2\})}{1+r} \right\} \qquad (\mathbf{P-3})$$

s.t.

$$-k + qf(q)\frac{y_2 - \omega}{r+\delta} \geq 0$$

An unemployed college worker searches for a routine job only if the realized cost of searching is too high. Let $\bar{c}$ be the cut-off, such that a worker searches for a routine job if the draw of the cost is greater than $\bar{c}$. The cut-off satisfies

$$V_2^2 - \bar{c} = V_2^1 \qquad (7)$$

Workers are indifferent between applying for a routine job versus searching a cognitive job while paying cost $\bar{c}$. The value of unemployment to a college worker $V(\{(b,0),2\})$ is then given by

$$V(\{(b,0),2\}) = b + F(\bar{c})[V_2^2 - \mathbb{E}(c|c<\bar{c})] + [1-F(\bar{c})]V_2^1 \qquad (8)$$

where $F(c)$ is the cumulative density function of c. An unemployed college worker searches *en-ante* for a cognitive job with probability $F(\bar{c})$. The return to search is V_2^2 and workers pay on average the expected cost $\mathbb{E}(c|c < \bar{c})$. With probability $1 - F(\bar{c})$, the worker searches for a routine job with return to search equals to V_2^1.

The last constraint optimization problem is about the unemployed high school workers. The problem is standard except that it is subject to an incentive compatibility constraint. Recall that routine jobs cannot post contracts conditional on workers' type, and college workers have potential incentives to apply for contracts hiring high school workers. Given that college workers quit endogenous, the same pair of wage and queue that generates zero expected profits from hiring high school workers would cost negative expected profits if hiring college workers instead. Therefore, in equilibrium, no employer posts contracts to high school workers that would also attract college workers. The incentive compatibility constraint specifies that offers made to high school workers do not attract college workers.

The value of an unemployed high school worker $V(\{(b, 0), 1\})$ is such that

$$V(\{(b,0),1\}) = b + \max_{\omega,q} \left\{ f(q)\frac{V(\{(\omega,1),1\})}{1+r} + [1-f(q)]\frac{V(\{(b,0),1\})}{1+r} \right\} \qquad (\mathbf{P-4})$$

s.t.

$$-k + qf(q)\frac{y_1 - \omega}{r+\delta} \leq 0$$

$$f(q)\frac{V(\{(\omega,1),2\})}{1+r} + [1-f(q)]\,\frac{V(\{(b,0),2\})}{1+r} \leq V_2^1 \qquad \text{(Incentive compatibility)}$$

where the value of a match for a high school worker satisfies:

$$\frac{V(\{(\omega,1),1\})}{1+r} = \frac{\omega}{r+\delta} + \frac{\delta}{r+\delta}\frac{V(\{(b,0),1\})}{1+r}$$

The incentive compatibility constraint is essential to understand the mechanism of displacement. The left-hand side of the equation is the expected return when a college worker apply in the market with (ω, q). Notice that it includes the value of on-the-job search in the future. The right-hand side of the equation is the highest value of college workers searching optimally for a routine job when unemployed, where employers anticipate that workers quit voluntarily. The incentive compatibility constraint states that college workers prefer to apply for jobs where employers expect them to search for better matches later, rather than mimicking high school workers. In equilibrium, the constraint might or might not be binding depends on parameter values. When it is not binding, the search decision of high school workers and college workers are separated. The allocations of high school workers are constraint efficient. When the constraint binds, the search decision of college workers affects the labor market outcomes of high school workers. And the unemployment of high school workers is distorted.

Notice that the problems constructed above imply that college workers and high school workers do not compete in the same market for routine jobs. Firms anticipate to meet with only one type of workers, and the market separates workers with different wages. Alternatively, one could imagine a structure, where routine employers attract a mixed distribution of workers and cross-subsidize college workers. In general, an allocation with this structure is difficult to be supported as an equilibrium. The intuition is as follows. Once matched with a routine job, the employment dynamics of college workers and high school workers are different. This means when searching from unemployment, the level of wage a college worker could give up for a marginal increase in matching rate, in general, are different from high school workers. In other words, if both types apply in the same market, they can always be separated by a pair of wage and queue such that one type is indifferent while the other type strictly prefer. A deviating allocation that attracts high school workers only always exists and is profitable without the cross-subsidy. In this paper, I focus on the separating equilibrium, where college and high school in different markets for routine jobs.

Given Proportion 1, the existence of an separating equilibrium relies on whether there is a solution to problems $(\mathbf{P-1})$ to $(\mathbf{P-4})$. The following proposition establishes the existence of a separating equilibrium

Proposition 2 *Assume that* $(y_2 - y_1)/(y_1 - b) < (1+r)/\delta$. *There exists a number* $\bar{k} > 0$ *such that for all* $k \in (0, \bar{k}]$, *there is a separating equilibrium. The allocation supported by the equilibrium is uniquely characterized in the Appendix.*

The assumption requires that the productivity gap between the two types of jobs, relative to the match surplus of routine jobs, should be bounded. It ensures a positive fraction of college workers applying for routine jobs when unemployed. In equilibrium, some college workers apply for routine jobs and continue to search for cognitive jobs once employed, while others search directly for a cognitive job and stay until the match is separated. High school workers search for routine jobs, with posted wages different from the ones where college workers apply. Even though contracts do not specify the type of a worker, workers are sorted into separated markets according to their education.

The sorting mechanism of the separating equilibrium is as follows. Given that a higher wage is associated with a lower matching rate, the market sorts unemployed workers according to their tolerance to remain unemployed. Unemployed workers trade off matching rates and wages, base on the value of their outside option from employment, as well as the option of on-the-job search in the future. Workers with a higher value of unemployment are willing to wait longer for a higher wage. This is the standard trade-off in search models. Workers with more chances switching to a better job, however, are willing to apply for a lower wage to be employed faster. If a worker can search on-the-job, the wage received out of employment consists only a fraction of the total value of the employment. The sooner the worker becomes employed, the sooner she can start searching for a higher wage. Comparing college workers with high school workers, they substitute matching probability and wage at different rates when searching for routine jobs. College workers have shorter expected job tenure because they use routine jobs a stepping stone towards better jobs. On the other hand, high school workers do not search on-the-job. Therefore, wage offered by the routine employed determines the entire income flows of their employment. For a given level of the outside option, high school workers are less willing to trade-off wages for matching rate, comparing with college workers.

The incentive compatibility constraint might or might not be binding in equilibrium. Below I show a sufficient condition such that the incentive compatibility constraint is binding and the equilibrium suffers from adverse selection.

Proposition 3 *There exist numbers $\bar{k}_1 > 0$, $\bar{\theta} > 0$ and $\bar{y} > y_1$ such that for all $k \in (0, \bar{k}_1)$, $\theta \in (0, \bar{\lambda})$ and $y_2 \in (y_1, \bar{y})$, the equilibrium suffers from adverse selection, and the unemployment rate of high school workers is inefficiently high.*

Consider the environment where unemployed college workers search only for routine jobs (when $\theta \to 0$), and receive all the match surplus once switch to a cognitive job (when the cost of posting poaching offers converges to zero). If the productivities of the two types of jobs equal, the total surplus of a match between a worker and a routine job are the same for both types of workers. The value of employment with a routine employer is also identical for all workers since firm's share of surplus is constant. College workers apply to markets with the same queue length as high school workers and receive on average the same wage during employment. However, they receive a low initial wage from routine employers and a high poaching wage from cognitive employers. Since the wage offered from a routine employer is lower for college workers, they have strict incentives to search in the same market as high school workers and apply for cognitive jobs later. Equilibrium with adverse selection exists for an open space in the three parameters as stated in Proposition 3. In section 4, I show with numerical simulations that the incentive compatibility constraint binds under large ranges of reasonable parameters.

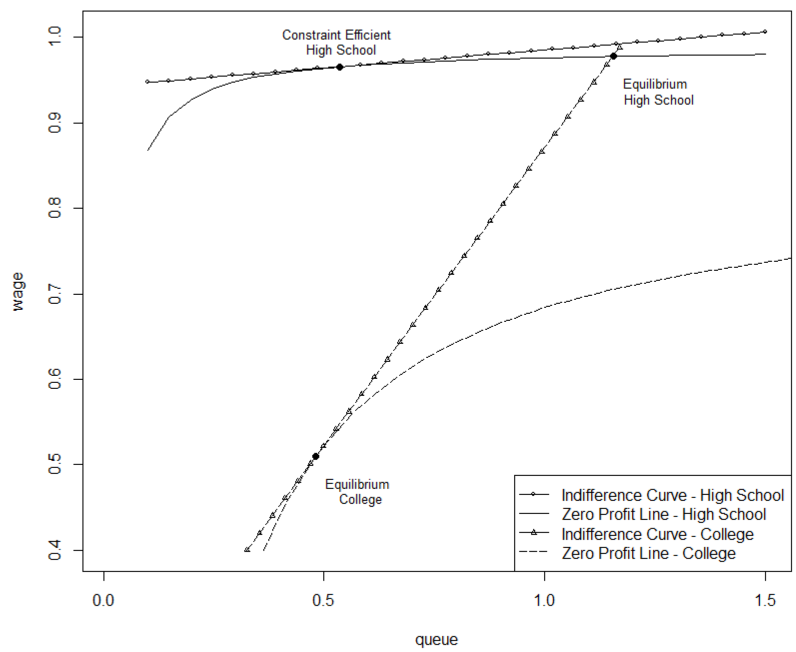
When the equilibrium suffers from adverse selection problem, the equilibrium allocation of high school workers is distorted. In particular, the unemployment rate of high school workers is higher than under the constrained efficient allocation. The intuition is that comparing with high school workers, college workers are less tolerant to unemployment when applying for routine jobs. In equilibrium, contracts offered to high school workers are associated with a job finding rate, low enough such that college workers are discouraged from applying.

Figure 2 demonstrates the mechanism in a wage-queue space.[12] The indifference curves of workers are upward sloping since workers prefer higher wages and lower queues. The slope is steeper for college workers since they would substitute more wage for a marginal increase in matching rate. The

[12] The figure is simulated under parameter set: $k = 0.2$, $y_1 = 1$ $y_2 = 4$, $r = 0.01$, $\delta = 0.1$, $\theta = 1$ and $M(u, v) = \frac{uv}{u + r}$.

zero-profit lines of employers are also upward sloping, given that employers prefer lower wages and longer queues. At any given level of matching rate, employers offer a lower wage to college workers to compensate their relatively shorter expected job tenure. When college workers are better-off applying to the market where high school workers are optimized, no employer searches that market in equilibrium. Instead, they entry market where college workers are indifferent from their equilibrium allocation while making zero expected profits hiring high school workers. Because high school workers trade-off wage to queue at a higher rate than college workers, the equilibrium allocation of high school workers is to the right of the constrained efficient allocation. The matching rate for high school workers is higher than the undistorted level, and their equilibrium unemployment rate is inefficiently low.

Figure 2: Equilibrium allocation in the Routine job market



The displacement of high school workers out of employment is a market solution to the adverse selection problem. In equilibrium, employers offer wages above efficient level, anticipating that market competition rises with an increase of wage. Workers with different career mobility sort into separated contracts because the levels of market competition associated with these contracts are different. When adverse selection is present, the cost of separating is born by high school workers only. Even though high school workers receive an inefficient high wage in equilibrium, their return to search and the sum of discounted total labor income as an unemployed worker are inefficiently low. The gains from distorted wage do not compensate the distortion on the matching rate from unemployment. In this sense, the model implies that the skill premium, measured as the wage differential between high school and college workers, underestimates the true returns to college education.

I interpret the separating equilibrium as sorting workers with different educational backgrounds into different occupations. Consider markets within the routine-job sector as different occupations with similar skill requirements and productivity, and are all involved mostly with routine based tasks. Employers post jobs from various occupations and anticipate that a high wage would attract more applicants. Within the routine-job sector, college workers are attracted to jobs in occupations that are easier to find, have higher turnover and are associated with a lower payment. Instead, high school workers prefer jobs in occupations that compensate better, have a lower turnover but generate more competition to be employed. As observed in the labor market, college workers do not search randomly in less skilled occupations. Among low-skilled occupations with large employment, the fraction of college graduates are high in occupations such as retail salespersons (24.6%), recreation attendants (23.5%) and bartenders (16.5%), occupations that usually pay a low wage and have high turnover rates.

Wages in the model represent the total compensation from a job. Higher wages in the model do not necessarily mean higher salaries. Even though the pay rates of two jobs are identical, the job that provides benefits and insurance is considered as paying a higher wage than the one that only pays salary. High school workers have limited career upgrades and stay on average longer than college workers with a routine employment. Additional compensation such as benefits and insurance are more attractive to them than college workers, who search for routine jobs as a stepping-stone in the career path.

Notice that the displacement of high school workers out of employment is inefficient. This is in sharp contrast to the literature that studies the crowding-out effect of over-qualified workers using a competitive framework (see Beaudry et al. (2013) and Acemoglu and Autor (2011)). In these papers, educated workers move down the job ladder in the scarcity of skilled jobs, and replace less-educated workers efficiently. These studies take as given that educated workers are more productive and are preferred to less-educated workers by the employers, even if the jobs are routine based and have less skill requirement. The dynamic mobility of overqualified workers are dismissed, and the crowding-out effect relies heavily on the absolute productivity advantage of educated workers. In other words, no externality is generated by mismatched workers, and the displacement of high school workers is an efficient productivity-driven adjustment. This paper, on the other hand, argues that the displacement of high school workers is a market distortion. The mechanism does not reply on whether college workers are more or less productive than high school workers in routine jobs.[13] The search behavior of college workers generates a negative spillover effect on the unemployment rate of high school workers. The distinction in welfare implications

[13] The mechanism functions if college workers are less productive than high school workers or more productive to a certain limit.

would lead to very different policy recommendations.

Another group of literature that studies the crowding-out effect uses frictional models with random matching (see Gautier (2002), Albrecht and Vroman (2002) Dolado et al. (2009)). These studies impose that educated workers and less-educated workers are pulled together and match randomly to employers offering low-skilled jobs. Educated workers create a congestion effect to the existing less-educated workers if they reduce the incentives of employers to create low-skilled jobs due to a higher quit rate. In contrast to this paper, these studies do not take into account that employers have incentives to separate workers, and workers have different trade-offs between the matching rate and the wage. Even without imposing direct competition, I show that high school workers could still be displaced out of employment because of adverse selection. Besides, this paper discusses the market distortion on high school workers alone both the intensive and extensive margin. The studies with random matching models have to impose assumptions on either the equilibrium wage or the market matching rate, to keep the model tractable in the presence of on-the-job search.

The market distortion arises because employment contracts of routine jobs cannot exclude overqualified college workers by offering wages conditional on workers' type. In general, there are many labor market implications that could potentially solve the problem. For example, employers could offer dynamic wage-tenure contracts with sufficiently back-loaded payments.[14] Also, if college workers are less tolerant to work long hours in routine jobs, employers could potentially exclude them using contracts with extensive hour requirement. How the displacement mechanism works under contracts with different extended structure is beyond the scope of this paper. Instead, I present in the next section the implication of the labor market distortion for educational choices, in particular, the demand for vocational based education.

4 Implications for educational choices

In this section, I discuss the implications of the displacement on the choice and return to education. In particular, I argue that post-secondary vocational education can be an institutional solution to the adverse selection problem. Vocational education can mitigate the market distortion and has a higher market value than it is commonly thought.

Vocational education (VE) here refers to the non-baccalaureate post-secondary level education that focuses primarily on providing occupationally specific preparation. In the U.S., post-secondary vocational education is provided mostly by community colleges, and students are awarded either

[14] Contracts with generous pension plans are examples of back-loaded wage-tenure contracts and were by many union-based jobs in the 70's. However, with the delined union rate, this type of contracts becomes less popular for routine jobs today.

a post-secondary certificate or an associates degree once completing the programs. In the United State, vocational education has been growing fast in the past few decades. As showed at the beginning of the paper, the total number of vocational based certificates awarded per year has increased by 60 percent from 2000 to 2010, growing much faster than the 34 percent increase of bachelor degrees.[15]

In this section, I first extend the baseline model to include educational choices. I show that vocational education has positive market value as an entry barrier to college workers. I also argue that, in contrast to most of the literature, introducing costly entry barriers such as vocational education and occupational licensing is welfare improving. I then perform a series of numerical simulations. I examine how the choice of education changes under some recent labor market changes that are frequently discussed in the literature. Examples include an increase in the return of cognitive skills, a decline in the return of routine jobs, as well as an increase in college tuition.

4.1 A labor market with educational choices

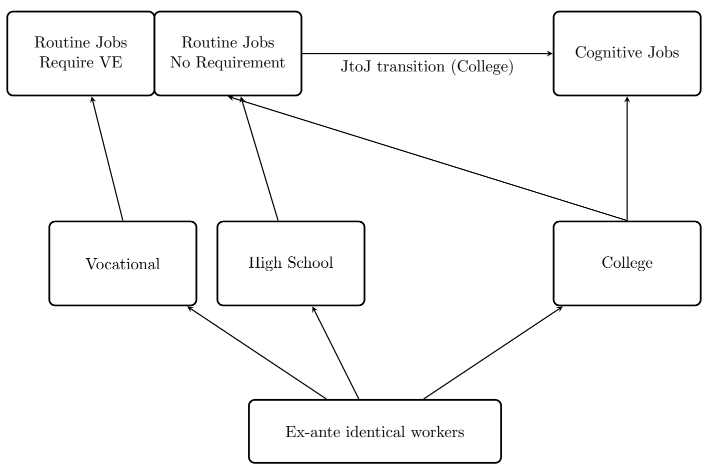
The distribution of workers' educational attainment is now endogenous. All workers are ex-ante identical. In period zero, an individual's education cost e is realized and drawn from a distribution with cumulative function $G(\cdot)$. The cost reflects the differences of individuals in both financial constraints and their abilities. An individual makes the education choice, based on her realized cost and the expected return of education from the labor market. An individual can choose either to hold only a high school diploma or pursuit post-secondary education. High school education is considered to be free in the model, while a workers with individual cost e needs to spend αe for college and βe for vocational education. The total cost of education varies across individuals. I assume that $\alpha > \beta$, in the sense that comparing with vocational education, a college education is more expensive, takes longer and requires more efforts. Education is a one-time choice, accomplished before entering the labor market. After period zero, all workers enter the labor market with either a high school diploma, a vocational credential or a bachelor degree.

There are now two types of routine jobs: ones that have no educational requirement and ones that require a vocational degree to enter. The former group is still called routine jobs while the latter group is referred to as vocational jobs (denoted as v). Now there are three types of jobs in the labor market: cognitive jobs, routine jobs, and vocational jobs. The cognitive jobs and routine jobs are the same as in the baseline model. The vocational jobs are also routine based, but they have barriers to entry. I assume that vocational jobs and routine jobs are equally productive such that $y_2 > y_1 = y_v$. The assumption is made to isolate the role of vocational education that mitigates market distortion from its value of providing human capital. In the numerical simulations presented later, this assumption will be relaxed.

[15] National Center for Education Statistics (NCES).

The labor market dynamics is depicted in Figure 3. The flow of high school workers and college workers are identical as in the baseline model. Their job search options are not affected by the presence of vocational jobs since vocational jobs have entry barrier. An unemployed worker with a vocational credential can apply for either routine jobs or vocational jobs. In equilibrium, the worker applies to vocational jobs only and do not search on-the-jobs once matched. To see this, notice that while the productivities of both jobs are equal, routine jobs sector might suffer from market distortions. Therefore, the return to search when applying for vocational jobs weakly dominates the return to search for routine jobs. Besides, workers with vocational education do not have the incentive to mimic high school workers and search on-the-job like college workers. No firm with vocational vacancy has the incentive to poach workers from routine jobs, given that the productivity of routine jobs and vocational jobs are equal.[16] For the same reason, once matched with a vocational job, there is no option value for the worker to search on-the-job.

Figure 3: labor Market Flows with Educational Choices



Denote by U_h, U_v, U_c the equilibrium return to search from unemployment for a high school worker, a worker with a vocational credential and a worker with a bachelor degree respectively. Since college workers cannot apply for vocational jobs, the equilibrium allocation of workers with vocational education is the constrained efficient allocation that a high school worker would receive if there is no displacement.

The educational choice of a worker before entering the labor market is determined as follows. Let $\bar{e}$ be the cutoff such that a worker with the educational cost greater than $\bar{e}$ does not pursue post-secondary education. The return to vocational education relative to high school diploma must

[16]If the productivity of vocational job is greater than routine jobs, workers with vocational education might have the incentive to mimic high school workers. However, when high school workers are displaced due to the incentives from college workers, workers with vocational education do not further distort the market. Therefore, there is no incentive for workers with vocational education to mimic high school workers.

equal the cost of vocational education with $\bar{e}$, such that

$$U_h = U_v - \beta \bar{e} \tag{9}$$

Similarly, let $\underline{e}$ be the cutoff such that a worker with educational cost lower than $\underline{e}$ enter university to obtain a bachelor degree. The cutoff $\underline{e}$ must satisfy:

$$U_v - \beta \underline{e} = U_c - \alpha \underline{e} \tag{10}$$

The demand for vocational education is positive if and only if $\bar{e} > \underline{e}$.

Demand for vocational education

When the economy suffers from adverse selection, the equilibrium market of routine jobs is distorted, and the labor market return to high school worker is inefficiently low. Unlike routine jobs, jobs that require vocational education have barriers to entry and can exclude college workers from seeking stepping-stone opportunities. Therefore employer with vocational jobs can offer non-distorted wages, even though the nature of vocational jobs is also routine based. By obtaining vocational education, workers gain access to markets that do not suffer from the adverse selection and could improve their labor market outcomes. The value of vocational education as an entry barrier arises preciously because of the existing displacement that distorts the labor market.

I formalize this idea in the next proposition. As long as the cost of vocational education is not too high, there exists a demand for vocational education when the labor market suffers from adverse selection.

Proposition 4 *When an equilibrium exists with adverse selection, there is a level $\bar{\beta} \in (0, \alpha)$ such that for any $\beta \in (0, \bar{\beta})$, the equilibrium has positive demand for vocational education.*

The gain from vocational education relative to high school diploma equals the amount of distortion on high school workers caused by the threat of overqualified college workers. The larger is the distortion; the higher is the return to vocational education. As a barrier to entry, vocational education mitigates adverse selection by excluding college workers from applying for routine jobs. In this sense, vocational education helps to sort routine jobs to the right workers. When the cost of taking vocational education does not exceed the gain, some individuals with cost realizations between $\underline{e}$ and $\bar{e}$ find it optimal to pursue vocational education.

When there is a demand for vocational education, ex-ante, an individual enters college with probability $G(\underline{e})$, chooses vocational education with probability $G(\bar{e}) - G(\underline{e})$, and graduates with only high school diploma with probability $1 - G(\bar{e})$. Let U_0 be the ex-ante utility of a worker at period 0, before the realization of individual's educational cost. The level of U_0 equals the expected return from the labor market net the expected costs paid to acquire education, such that

$$U_0 = G(\underline{e})U_c + [G(\bar{e}) - G(\underline{e})]U_v + [1 - G(\bar{e})]U_h - \beta\mathbb{E}[e|\underline{e} < e \leq \bar{e}] - \alpha\mathbb{E}[e|e \leq \underline{e}]$$

Since all workers are identical ex-ante and all employers in equilibrium make zero expected profits, I can rank the total welfare of different economies by comparing the levels of U_0. Consider the following two economies with the following education systems: one is as described above where workers can choose from high school, vocational education or college education, while the other one has the college education as the only option for post-secondary education, with the cost rate α. By comparing the ex-ante utility of workers in both economies, I show in the following proposition that even without productivity premium, costly vocational education can improve welfare, given that the labor market suffers from adverse selection.

Proposition 5 *When an equilibrium exists with adverse selection, the option to undertake vocational education makes workers better off ex-ante, for* $\beta \in (0, \bar{\beta})$.

Comparing with an economy with a positive demand of vocational education, the economy without the option of vocational education has more high school workers and also more college workers. The high school workers who would otherwise be attracted to vocational education could gain from a better labor market outcome when the adverse selection problem is present. Vocational credentials provide them the access to jobs that exclude college workers from distorting the market. The college workers who would otherwise choose vocational education could gain from the costs saved from taking less costly education. These workers have relatively higher costs of taking post-secondary education than other college students. Their labor market return net the cost of education is relatively smaller. With the option of vocational education, these workers are better off even though the return to vocational education is lower than the return to college, the cost they could save from pursuing vocational education surpasses the difference in returns. When the adverse selection problem is present, introducing vocational to the labor market strictly improves welfare, as long as the cost of vocational education is not too large relative to obtaining a bachelor degree.

Notice that this result is very different from the standard literature with competitive markets. In a competitive market, a barrier to entry creates monopoly power and distorts efficiency. When low skilled workers are displaced efficiently by over-qualified workers, excluding college workers from applying some routine-based jobs would reduce the overall welfare of the economy. When the displacement of high school workers happens through adverse selection, on the contrary, vocational education as a barrier to entry has an important benefit to the labor market. It improves market efficiency by sorting some workers into non-distorted markets and increases the overall welfare of the economy.

Occupational licensing

Occupational licensing is often considered as a barrier to restrict entry to certain professions. According to Kleiner and Krueger (2013), nearly 30 percent of U.S. workers with more than high school education, but not a bachelor degree, were required to hold a license on their jobs. Comparing with occupational certification and registration, licensing is the toughest form of occupational regulation. Candidates need to pay a series of fees and pass exams that could be written or practical or both. This process usually takes weeks and sometimes several months. For intermediate-skill occupations, there is a large group that require licensing but obtaining a license does not require long periods of training and skill development. Examples include transport drivers such as truck drivers and bus drivers, child carers, security personnel, miners, earth-moving plant operators, and controlled personal service workers.

One concern of occupational licensing is that as an entry barrier, occupational licensing may cause job losses by increasing employment costs (Kleiner, 2005). This concern is particularity relevant for occupations that are licensed for conformance. Unlike the high-skill licensing systems, conformance licensing that mostly exists in the middle- and low-skill occupations, has little emphases on the development of skill and does not create an occupational monopoly (Cooney, 2013). With little gain from human capital development, the monetary and time investment to obtain a license incurs a cost barrier to entry in these occupations.

The above-discussed mechanism of how a barrier to entry in some occupations could improve labor market applies to occupational licensing as well. Imposing occupational licensing, particularly in the low- and intermediary skilled occupations create a costly barrier that can help exclude overqualified college workers from seeking stepping-stone opportunities. In this sense, occupational licensing helps to sort workers with different educational background into jobs that match their skill levels. Occupational licensing can have an important benefit to employment, because it provides protections for middle and low skilled worker from the competition of over-qualified college workers. Imposing occupational licensing in some occupations can increase the employment opportunities of these workers with an increase in market efficiency.

4.2 Labor market trends and post-secondary education

In this section, I simulate the model to understand the effects of some key labor market trends on the attainment of post-secondary education. In particular, I consider a positive change on the return to cognitive skills, an adverse change on the return of routine tasks as well as an increase in the cost of college education. The first two changes are often referred to as the skill-biased technological changes (SBTC), in the sense that the process of automation and offshoring complements skilled cognitive jobs but substitutes less-skilled routine jobs over the past thirty years.[17] I also discuss

[17] See Autor et al. (2003) on routine jobs, Goos et al. (2014) on skill-biased technological change versus globalization and Autor et al. (2014) on exposure to international trade.

the impact of an increase in college tuition on education attainment under the two labor market trends.

Benchmark Parameter Values

The matching technology has the form $M(u,v) = \frac{uv}{u+r}$. Time is measured in quarters. The values of parameters are as follows. The productivity of routine jobs y_1 is normalized to 1. The productivity of cognitive jobs y_2 equals 2.5 while vocational jobs have productivity $y_v = 1.5$. The flow income of unemployment b is set to be 0.2. The risk-free return rate $r = 0.01$, which is equivalent to an annual discount rate of 0.96. Since the model has endogenous job separation from on-the-job search, the exogenous separation rate is set to be $\delta = 0.05$, consistent with the finding of Sahin et al. (2010). The cost of posting vacancies k equal to 0.2. I assume the search cost c follows exponential distribution with parameter $\theta = 1$. For all the simulation examples below including the benchmark case, the incentive compatibility constraint always binds and high school workers have distorted labor market outcomes.

Individuals cost of educational e follows Beta distribution with parameter $(4, 4)$. The educational cost rate for college and vocational training are set to be $\alpha = 450$ and $\beta = 135$ so that the educational attainment matches roughly the distribution of educational attainment in the 1970s.[18]

Table 2 presents the simulation outcomes, including the distributions of educational attainment, the unemployment rates and wages of workers across educational groups. Column 2 is the benchmark economy. The economy has 80 percent high school graduates and only 10 percent college graduates. Workers with more education have higher wage and lower unemployment rate. The unemployment rate for high school workers is around 10 percent, while the unemployment rate for workers with post-secondary education is less than seven percent. The college-versus-high-school wage ratio is 2.48, matching the college-versus-high-school earnings ratio in the early 80s (Acemoglu and Autor, 2011). About 13 percent of college workers search for routine jobs when unemployed.

To see how large is the distortion caused by adverse selection, I compute the constrained efficient allocation of high school workers. If the adverse selection is absent, the non-distorted labor market outcome of high school workers has 7.1% rate of unemployment and wage equals to 0.97. The distortion from overqualified college workers raises the unemployment rate of high school worker for 38 percent, with only one percent increase in the wage, compared with the constrained efficient allocation of high school workers. The distortion on the wage is tiny relative to the distortion on the unemployment rate.

[18]U.S. Bureau of Labor Statistics. I use the group of workers with some college as well as workers with associate degrees in the data to proximate vocational workers in the model.

Table 2: Comparative Statics

	Benchmark	Scenario 1	Scenario 2	Scenario 3
Education Attainment				
College	10%	98%	10%	33%
Vocational	10%	0	52%	27%
High School	80%	2%	38%	40%
Unemployment Rate				
College	6.1%	5.6%	6.0%	5.6%
Vocational	6.6%	0	6.6%	6.6%
High School	9.7%	10%	10.7%	11.1%
Mean Wage				
College	2.43	4.92	2.43	4.92
Vocational	1.46	0	1.46	1.46
High School	0.98	0.98	0.68	0.68

Increased productivity of cognitive jobs

In this exercise, I double the productivity of cognitive jobs such that cognitive jobs are five times productive as routine jobs. The results are presented in the third column of table 2 under Scenario 1. Increasing productivity of cognitive jobs while keeping the educational cost unchanged, more workers are attracted to the college education. In this example, 98 percent of workers choose to be a college graduate. Comparing with the benchmark economy, the wage of college workers doubled along with a five percent decline in unemployment rate. The net return to college education becomes much higher so that no workers choose vocational education and only two percent of workers are high school graduates.

The positive shock to cognitive jobs also affects the labor market outcomes of high school workers through the mechanism of adverse selection. The return to search for high school workers declines as adverse selection become more server. When the return to cognitive skills increases, the value of unemployment for college worker also increases. An unemployed college worker becomes more "patient" when applying for routine jobs. The reason is that if the job search fails, the worker has the chance to take another drawing of the search cost in the next period and might end up searching for a cognitive job instead. As a result, college workers are less willing to accept a routine job with a low wage when the value of cognitive jobs is higher. To exclude college workers from applying, employers further distort the wage offered to high school workers causing lower job finding rates. This further distortion on high school workers is reflected in the increase in the unemployment rate and a negligible wage rise. Overall, the labor market return of high school workers declines which increase the incentive of workers to attain post-secondary education.

Decreased productivity of routine jobs

Now consider a fall on the productivity of routine jobs by 30 percent so that $y_1 = 0.7$. This negative change on routine jobs the can be considered as the automation and offshoring process that either substitute or offshore routine based tasks. Scenario 2 in table 2 present the results. The negative shock reduces the return to search to an unemployed high school worker. The wage of high school workers declines by 30 percent, proportional to the change in productivity, and the unemployment rate increases by 10 percent. The negative shock on routine jobs also affects the return to search for college workers, but the magnitude of the effect is tiny. When routine jobs become less productive, college workers are more likely to search for cognitive jobs when unemployed. With a declined value of outside option, they apply to cognitive jobs that have relatively lower wage offer and high job finding rate, when searching for cognitive jobs. The negative shock reduces the overall unemployment rate of college workers and also lowers their average wage. Although college workers are less "mismatched", their overall return to the labor market falls.

With a fall in the value of a high school diploma, high school graduate have more incentives to attain post-secondary education. The increase in post-secondary education attainment, however, is captured almost entirely by a 42 percent increase in vocational education. Even though workers with a college education earn 65 percent more than workers with vocational education and have 10 percent lower unemployment, the college education is also three times more costly than vocational education. The premium of college education in this example is not significant enough to attract more individuals.

Increased college tuition

According to the NCES, the college tuition of both public and private institution have increased more than twofold since the 1980's. In this exercise, I combine the two shocks discussed above and I also include an increase in the educational cost rate for college, such that α is doubled. As shown in the last column of Table 2 under Scenario 3, the economy has 33 percent college workers, 27 percent vocational workers, and 40 percent high school workers, similar to the educational attainment in the U.S. in 2010. The relative wage ratio between college workers high school workers is 7.2 and the wage ratio between college workers and vocational workers is 3.3. The unemployment rate of workers with vocational education is one percentage point higher than college graduates, while the unemployment rate of high school workers is twice as high as the college graduates. The increased productivity on college jobs attracts workers to enter colleges, while the negative shock on routine jobs reduces the incentives for workers to hold high school diploma only. The change in educational costs distributes workers into colleges versus vocational education.

I relate these results to the argument of Autor et al. (2010) who points out that one puzzle in the U.S. labor market is that given the steep rise in the college-versus-high-school earnings ratio, the relative supply of college-educated workers is not growing fast enough. This paper suggests that some of the increase in post-secondary education has taken the form of non-bachelor level post-secondary vocational education. Because skill-biased technological changes have increased not only the premium of college graduates relative to high school graduates, it also affects the relative return between non-university vocational education verse high school education. Together with an increase in college tuition and costs, vocational education becomes another attractive educational investment among students.

5 Conclusion

In this paper, I argue that the labor market generates inefficient unemployment of high school graduates as a mechanism to separate high school graduates from overqualified college graduates searching for some types of routine jobs. This is how the labor market resolves the adverse selection problem arising from the fact that employment contracts in those routine jobs do not discriminate between high school and college graduates. In this context, the demand for vocational education arises because it allows employers to exclude applicants who treat the job as a stepping-stone. As a result, a vocational credential provides high school workers with access to markets that do not suffer from the distortion of over-educated workers and improve their labor market outcomes. Costly entry barriers on some occupations can improve labor market efficiency by sorting the right workers to routine-based jobs.

The model helps understand the effects of skill-biased technological change on the increase of post-secondary education in the past thirty years. Using numerical simulations, I show that skill-biased technological changes increase the return to post-secondary education at both university level and non-university level in the form of vocational education. Together with an increase in college tuition, skill-biased technological changes can explain both the increase in college education and post-secondary vocational education over the past thirty years.

Appendix

Proof of Proposition 1

Consider separating equilibrium only. To show statement i), consider an allocation that does not solve problems $(\mathbf{P-1})$ to $(\mathbf{P-4})$. Then there must exist a deviating contract and a market queue, constructed from the solution of problems $(\mathbf{P-1})$ to $(\mathbf{P-4})$, such that the posted contract generate on-negative profits of firms while some workers are strictly better off. This violates the definition of a refined equilibrium. Therefore, any allocation that does not solve problems $(\mathbf{P-1})$ to $(\mathbf{P-4})$ can not be an allocation of a equilibrium. Statement i) must be true.

To show statement ii), suppose that a solution to problem $(\mathbf{P-1})$ to $(\mathbf{P-4})$ exists. It is straight forward to construct the equilibrium contracts X^* and equilibrium states S^* based on the the optimal wages from the solution. To construct an equilibrium queue mapping and a belief function, consider the following example of Q^* and $\mu(|x)$. For cognitive-job sector, the belief is degenerated such that firms expect to meet only college workers. The queue mapping is such that firms earn zero expected profits. For routine-job sector, firms expect to meet with college workers only for wages posted that are less than the optimal wage of high school workers, and meet with high school workers only for wages posted equal or higher than the optimal wage of high school workers. The Q^* is such that firms makes zero expected profits for any wage less or equal to the optimal wage of college workers and any wage higher than the optimal wage of highs school workers. For markets with wages in between the above range, the corresponding queue length is fixed at the level of the optimal queue that solves the high school workers' problem. (figure) This particular queue mapping and belief function satisfy equilibrium condition (i) to (iii), such that workers optimize given Q^*, firms optimize given Q^* and $\mu(|x)$ and earn zero profits at S^*. The belief in equilibrium is consistent with the actual distribution. Finally, using the optimal market queues (job arrival rates) from the solution, one can construct the stationary distribution of workers by equating the inflow and outflow of equilibrium states as stated in condition iv.

Proof of Proposition 2

Given *Proposition 1*, it is enough to show that a solution exists to problems $(\mathbf{P-1})$ to $(\mathbf{P-4})$. Also notice that the search problems of college workers are independent from the search problems of the high school workers. So I first show that a solution exists to the problems $(\mathbf{P-1})$ to $(\mathbf{P-3})$ of college workers, and then show that there is always a solution that solves problem $(\mathbf{P-4})$ of high school workers.

College workers

Consider first the on-the-job search problem $(\mathbf{P-3})$. The first order conditions for an interior solution are given by

$$\lambda q = 1$$

where λ is the relevant Lagrange multiplier and

$$\frac{V(\{(\omega',2),2\})}{1+r} - \frac{V(\{(\omega,1),2\})}{1+r} = \lambda q \frac{1-\eta(q)}{\eta(q)} \frac{y_2-\omega'}{r+\delta}$$

Denote $\{\omega^e(\omega), q^e(\omega)\}$ a solution to the problem.

Lemma 1 *For any $\omega \in [0,\, y_1]$, $\{\omega^e(\omega), q^e(\omega)\}$ is given uniquely by the pair (ω', q) with $y_1 \leq \omega' < y_2$ and $0 < q_a \leq q' \leq q_b < \infty$ such that*

$$q' f(q') \left(\frac{y_2 - \omega'}{r+\delta} \right) = k$$

$$\frac{\omega' - \omega}{r + \delta + (1-\delta) f(q')} \geq \frac{1-\eta(q)}{\eta(q)} \frac{k}{q' f(q')}$$

and $q' \geq q_a$ with complementary slackness, where q_a is given by

$$q_a f(q_a) \left(\frac{y_2 - y_1}{r+\delta} \right) = k$$

and $q_b > q_a$ is given by

$$\frac{y_2 - y_1}{r+\delta} = \frac{k}{q_b f(q_b)} \left(1 + \left(\frac{1-\eta(q_b)}{\eta(q_b)} \right) \left(\frac{r+\delta+(1-\delta)f(q_b)}{r+\delta} \right) \right)$$

Proof: the first condition stated in the lemma comes from the non-negative profit constraint. The second condition follows from combining the two first order conditions, substituting $V(\{(\omega,1),1\})$ and $V(\{(\omega',2),2\})$ with the zero profit constraint, such that

$$\frac{V(\{(\omega',2),2\})}{1+r} - \frac{V(\{(\omega,1),2\})}{1+r} = \frac{\omega' - \omega}{r+\delta+(1-\delta)f(q')}$$

The solution is interior, i.e. $\omega' \geq y_1$ if and only if $q' \geq q_a$. Notice that the assumption $(r+\delta)k < y_2 - y_1$ ensures that $q_a < \infty$. Combining the two conditions in the lemma implies

$$y_2 - \omega - \frac{k}{q' f(q')} \left(r + \delta + \frac{1-\eta(q')}{\eta(q')} \left[r+\delta+(1-\delta) f(q') \right] \right) = 0$$

where q' increases monotonically with ω given that $qf(q)$ and $\eta(q)$ are increasing function while $f(q)$ is a decreasing function. q_b is defined such that $\omega = y_1$. The interior solution $q^e(\omega)$ is the unique value of q' that solves the above equation with $q_a \leq q' \leq q_b$. It is clear that $\infty > q_b > q_a$. *QED*

Invert the above equation such that the current wage ω is a function of the future q'

$$W(q') = y_2 - \frac{k}{q' f(q')} \left(r + \delta + \frac{1-\eta(q')}{\eta(q)} \left[r+\delta+(1-\delta) f(q') \right] \right)$$

for all $q_a \leq q \leq q_b$. Substituting $W(q')$ into $V(\{(\omega,1),2\})$, one can express the value of a routine job to college workers as a function of q'

$$\frac{\hat{V}(q')}{1+r} \equiv \frac{V(\{(W(q'),1),2\})}{1+r} = \frac{y_2}{r+\delta} - \frac{k}{q'f(q')\eta(q')} + \frac{\delta}{r+\delta}\frac{V(\{(b,0),2\})}{1+r}$$

Note that

$$\frac{d}{dq}\left(\frac{\hat{V}(q)}{1+r}\right) = \frac{k}{qf(q)}\left(\frac{\eta'(q)}{\eta^2(q)} + \frac{1}{q}\left(\frac{1-\eta(q)}{\eta(q)}\right)\right)$$

which is positive for any $q \in [q_a, q_b]$ given that $\eta(q) < 1$ and $\eta'(q) > 0$, and decreasing if $\eta'(q)/\eta^2(q)$ is a decreasing function, which is given by the concavity assumption of η. Therefore $\hat{V}(q)$ is a strictly concave function of q.

Let $S(s)$ be the net surplus of a match, denoted by the state s of the worker employed in the match. Define $\hat{S}(q')$ such that

$$\frac{\hat{S}(q)}{1+r} \equiv \frac{S(\{(W(q),1),2\})}{1+r} = \frac{y_1 - W(q)}{r+\delta+(1-\delta)f(q)} + \frac{V(\{(W(q),1),2\})}{1+r} - \frac{V(\{(b,0),2\})}{1+r}$$

The first term to the right of the second equation is the firm's share of surplus and the two remaining terms sums to the worker's share of surplus.

Lemma 2 *$S(\{(W(q),1),2\}) - V(\{(W(q),1),2\})$ is a strictly decreasing and convex function of $q \in [q_a, q_b]$. $S(\{(W(q),1),2\}$ is a concave function of $q \in [q_a, q_b]$ and is maximized at $q = q_b$.*

Proof:

$$\frac{d}{dq}\left(\frac{\hat{S}(q)}{1+r} - \frac{\hat{V}(q)}{1+r}\right) = \frac{(1-\delta)f'(q)}{[r+\delta+(1-\delta)f(q)]^2}\left(y_2 - y_1 - \frac{k(r+\delta)}{qf(q)}\right)$$
$$- \frac{k}{qf(q)}\left[\frac{1-\eta(q)}{q}\left(\frac{1-\eta(q)}{\eta(q)} + \frac{r+\delta}{r+\delta+(1-\delta)f(q)}\right) + \frac{\eta'(q)}{\eta^2(q)}\right]$$

The first term to the right of the equation is non-positive since $f'(q) < 0$ and $qf(q)(y_2 - y_1) \geq k(r+\delta)$ for $q \geq q_a$. The second line is negative given $\eta(q) < 1$ and $\eta'(q) > 0$. Hence, $\hat{S}(q) - \hat{V}(q)$ is a strictly decreasing function of q for $q \geq q_a$. Also, the first term to the right of the equation is increasing given that $f(q)$ is decreasing and convex and $qf(q)$ is increasing. The second line is increasing since $\eta'(q)/\eta^2(q)$ is decreasing with q. Therefore, $\hat{S}(q) - \hat{V}(q)$ is strictly convex.

To show that $S(\{(W(q),1),2\}$ is concave, differentiate $S(\{(W(q),1),2\}$ gives

$$\frac{d}{dq}\left(\frac{\hat{S}(q)}{1+r}\right) = \frac{(1-\delta)}{q^2[r+\delta+(1-\delta)f(q)]^2}\left(k(1-\eta(q)) - \frac{(r+\delta)\eta(q)}{r+\delta+(1-\delta)f(q)}\left[qf(q)\frac{y_2-y_1}{r+\delta} - k\right]\right)$$

the right hand side of the equation is strictly decreasing in q, it equals to zero if

$$\frac{y_2 - y_1}{r+\delta} = \frac{k}{qf(q)}\left(1 + \left(\frac{1-\eta(q)}{\eta(q)}\right)\left(\frac{r+\delta+(1-\delta)f(q)}{r+\delta}\right)\right)$$

which holds at $q = q_b$. *QED*

Now consider the off-job search problem $(\mathbf{P-2})$. The objective function in $(\mathbf{P-2})$ us not generally concave in $\{\omega, q\}$, because the current wage affects workers' future quit rates (q^e). Instead of solving the original problem, consider the following transformed problem

$$V_2^1 = \max_{q',q}\left\{f(q)\frac{V(\{(W(q'),1),2\})}{1+r} + [1-f(q)]\frac{V(\{(b,0),2\})}{1+r}\right\} \qquad (\mathbf{P-2'})$$

s.t.

$$-k + qf(q)\left[\frac{S(\{(W(q'),1),2\})}{1+r} - \frac{V(\{(W(q'),1),2\})}{1+r} + \frac{V(\{(b,0),2\})}{1+r}\right] \leq 0$$

$$q_a \leq q'$$

The first order conditions for an interior solution of problem $(\mathbf{P-2'})$ are given by

$$\frac{V(\{(W(q'),1),2\})}{1+r} - \frac{V(\{(b,0),2\})}{1+r} = \lambda q\left[\frac{1-\eta(q)}{\eta(q)}\right]\frac{k}{qf(q)}$$

and

$$\lambda q = -\frac{\frac{d}{dq'}\left(\frac{V(\{(W(q'),1),2\})}{1+r}\right)}{\frac{d}{dq'}\left(\frac{S(\{(W(q'),1),2\})}{1+r} - \frac{V(\{(W(q'),1),2\})}{1+r} + \frac{V(\{(b,0),2\})}{1+r}\right)} \qquad (11)$$

Given that $V(\{(W(q'),1),2\}$ and $V(\{(W(q'),1),2\}$ are functions of q', λq can be expressed as a function of q'. Substituting $V(\{(W(q'),1),2\})$ into the first order condition:

$$\frac{y_2}{r+\delta} - \frac{k}{q'f(q')\eta(q')} - \frac{r}{r+\delta}\frac{V(\{(b,0),2\})}{1+r} = \lambda q\left[\frac{1-\eta(q)}{\eta(q)}\right]\frac{k}{qf(q)}$$

Denote (q^e, q_2^1) the solution to problem $(\mathbf{P-2'})$, the value of unemployment $V(\{(b,0),2\})$ can be expressed using the above equation as

$$\frac{V(\{(b,0),2\})}{1+r} = \frac{1}{r}\left(y_2 - \frac{k(r+\delta)}{q^e f(q^e)\eta(q^e)} - \lambda q\left[\frac{1-\eta(q_2^1)}{\eta(q_2^1)}\right]\frac{k(r+\delta)}{q_2^1 f(q_2^1)}\right) \qquad (12)$$

Substituting this back to the first order condition, the optimal return to search for a routine jobs V_2^1 to a college worker is

$$V_2^1 = \lambda q_2^1\left(\frac{1-\eta(q_2^1)}{\eta(q_2^1)}\right)\frac{k}{q_2^1} + \frac{1}{r}\left(y_2 - \frac{k(r+\delta)}{q^e f(q^e)\eta(q^e)} - \lambda q_2^1\left[\frac{1-\eta(q_2^1)}{\eta(q_2^1)}\right]\frac{k(r+\delta)}{q_2^1 f(q_2^1)}\right) \qquad (13)$$

Consider the last problem of college workers, i.e. the off-job search problem $(\mathbf{P-3})$. The relevant first order conditions for an interior solution of the problems are

$$\lambda q = 1$$

and

$$\frac{V(\{(\omega,2),2\})}{1+r} - \frac{V(\{(b,0),2\})}{1+r} = \lambda q \frac{1-\eta(q)}{\eta(q)} \frac{k}{qf(q)}$$

Combining the first order conditions and substituting $V(\{(\omega,2),2\})$ gives

$$\frac{y_2}{r+\delta} - \frac{r}{r+\delta}\frac{V(\{(b,0),2\})}{1+r} = \frac{k}{qf(q)\eta(q)}$$

Denote the solution to the problem (ω_2^2, q_2^2). The above equations can be expressed as

$$\frac{V(\{(b,0),2\})}{1+r} = \frac{1}{r}\left[y_2 - \frac{k(r+\delta)}{q_2^2 f(q_2^2)\eta(q_2^2)}\right] \tag{14}$$

Substitute $V(\{(b,0),2\})$ back to V_2^2 gives

$$V_2^2 = \frac{1-\eta(q_2^2)}{\eta(q_2^2)}\frac{k}{q_2^2} + \frac{1}{r}\left[y_2 - \frac{k(r+\delta)}{q_2^2 f(q_2^2)\eta(q_2^2)}\right] \tag{15}$$

The equilibrium cut-off $\bar{c}$ follows

$$\bar{c} = V_2^2 - V_2^1 = f(q_2^2)\left[\frac{V(\{(\omega,2),2\})}{1+r} - \frac{V(\{(b,0),2\})}{1+r}\right] - f(q_2^1)\left[\frac{V(\{(W(q'),1),2\})}{1+r} - \frac{V(\{(b,0),2\})}{1+r}\right]$$

substitute the first order conditions from problem $(\mathbf{P-2'})$ and $(\mathbf{P-3})$

$$\bar{c} = \frac{1-\eta(q_2^2)}{\eta(q_2^2)}\frac{k}{q_2^2} - \lambda q_2^1\left[\frac{1-\eta(q_2^1)}{\eta(q_2^1)}\right]\frac{k}{q_2^1} \tag{16}$$

The value of unemployment to a college worker is given by

$$V(\{(b,0),2\}) = b + [1-F(\bar{c})]V_2^1 + F(\bar{c})[V_2^2 - \mathbb{E}(c|c<\bar{c})]$$

Denote $\nu(\bar{c}) \equiv F(\bar{c})[\bar{c} - \mathbb{E}(c|c<\bar{c})]$. Given that $F(\cdot)$ follows exponential distribution, $\nu(\bar{c}) = \bar{c} - F(\bar{c})/\theta$. Rewrite $V(\{(b,0),2\})$ as

$$V(\{(b,0),2\}) - b = V_2^2 - \bar{c} + \nu(\bar{c}) = V_2^1 + \nu(\bar{c})$$

substituting V_2^2 and $V(\{(b,0),2\})$ from equation (15) and equation (14) into the above equation gives

$$\frac{y_2 - b}{r+\delta} + \frac{\bar{c} - \nu(\bar{c})}{r+\delta} = \frac{k}{q_2^2 f(q_2^2)\eta(q_2^2)} + \frac{1-\eta(q_2^2)}{\eta(q_2^2)}\frac{k}{(r+\delta)q_2^2} \tag{$Eq-1$}$$

Similarly, substitute V_2^1 and $V(\{(b,0),2\})$ from equation (13) and equation (12) gives

$$\frac{y_2 - b}{r+\delta} - \frac{\nu(\bar{c})}{r+\delta} - \frac{k}{q^e f(q^e)\eta(q^e)} = \lambda q_2^1 \left(\frac{1-\eta(q_2^1)}{\eta(q_2^1)}\frac{k}{q_2^1}\right)\left[\frac{1}{r+\delta} + \frac{1}{f(q_2^1)}\right] \tag{$Eq-2$}$$

Finally, the zero profit condition for routine job employers hiring college workers in problem $(\mathbf{P-2'})$, stated as a function of q', is given by

$$\frac{k}{q_2^1 f(q_2^1)} = \frac{r+\delta}{[r+\delta+(1-\delta)f(q^e)]}\left[\frac{k}{q^e f(q^e)} - \frac{y_2 - y_1}{r+\delta}\right] + \frac{k}{q^e f(q^e)}\left(\frac{1-\eta(q^e)}{\eta(q^e)}\right) \tag{$Eq-3$}$$

The solution to the problems $(\mathbf{P-1})$, $(\mathbf{P-2'})$ and $(\mathbf{P-3})$ are a set of $\{q_2^1, q_2^2, q^e, \omega^e, \omega_2^1, \omega_2^2\}$ such that: $\{q_2^1, q_2^2, q^e,\}$ must solve the above three equations $(Eq-1)$ to $(Eq-3)$, with $\bar{c}$ given by equation (16); $\{\omega^e, \omega_2^1, \omega_2^2\}$ are characterized by the following equations:

$$\omega^e = y_2 - \frac{k(r+\delta)}{q^e f(q^e)} \tag{17}$$

$$\omega_2^2 = y_2 - \frac{k(r+\delta)}{q^e f(q^e)} \tag{18}$$

$$\omega_2^1 = y_1 - \frac{k\left[r+\delta+(1-\delta)f(q^e)\right]}{q_2^1 f(q_2^1)} \tag{19}$$

I now show that a solution to the problems exists with the following steps. First I show that for a given $\bar{c}$, a solution exists to equations $(Eq-1)$ to $(Eq-3)$. Next I construct $D(\bar{c}) = V_2^2 - V_2^1$ using the values of $\{q_2^1, q_2^2, q^e\}$ implied with a fixed $\bar{c}$ and show that a fixed point $D(c_0) = c_0$ exists, with $c_0 > 0$.

Lemma 3 *For any given $\bar{c} \in [0, c_b]$, a solution to equation $(Eq-1)$ to $(Eq-3)$ exists with $q^e(\bar{c}) \in (q_c(\bar{c}), q_d)$, where $q_c(\bar{c})$ solves the following equation of q_c*

$$\frac{y_2 - b}{r+\delta} - \frac{\nu(\bar{c})}{r+\delta} = \frac{k}{q_c f(q_c)\eta(q_c)}$$

and q_d is such that

$$\frac{r+\delta}{r+\delta+(1-\delta)f(q_d)}\left[\frac{k}{q_d f(q_d)} - \frac{y_2 - y_1}{r+\delta}\right] + \frac{k}{q_d f(q_d)}\left(\frac{1-\eta(q_d)}{\eta(q_d)}\right) = k$$

and c_b is such that $q_c(c_b) = q_d$.

Proof: Consider first the equation $(Eq-1)$. The right hand side of the equation is a decreasing function of q_2^2. If q_2^2 converges to 0, the right hand side converges to ∞ and if q_2^2 converges to ∞, it converges to $k \leq (y_2 - b)/(r+\delta)$. Since $\bar{c} - \nu(\bar{c}) \geq 0$ for $\bar{c} \geq 0$ and $\lim\limits_{\bar{c}\to\infty}(\bar{c} - \nu(\bar{c})) = 1/\theta < \infty$, therefore, for any $\bar{c} \geq 0$ there exists a unique $q_2^2 < \infty$ such that equation $(Eq-1)$ holds.

Now consider equation $(Eq-2)$. First, differentiate λq_2^1 and notice that $\partial \lambda q_2^1/\partial q^e < 0$ if and only if $\hat{V}''(q)\hat{S}'(q) > \hat{V}'(q)\hat{S}''(q)$. This condition holds since $\hat{S}(q) - \hat{V}(q)$ is decreasing and convex, therefore $\hat{S}'(q) < \hat{V}'(q)$ and $\hat{S}''(q) > \hat{V}''(q)$.

Given that $\partial\lambda q_2^1/\partial q^e < 0$, equation $(Eq-2)$ characterizes q_2^1 as a declining function of q^e. If q_2^1 converges to 0, the right hand side of the equation converges to infinity and if q_2^1 converges to infinity, it converges to 0. Let $q_c(\bar{c})$ be the unique level of q^e such that the left hand side of the equation equals to zero for any given $\bar{c}$, i.e.

$$\frac{y_2 - b}{r+\delta} - \frac{\nu(\bar{c})}{r+\delta} = \frac{k}{q_c f(q_c)\eta(q_c)}$$

The right hand side of the above equation is a decreasing function of q_c. As q_c converges to zero, the right hand side of the equation converges to infinity and when q_c converges to infinity, it converges to k. Define c_a such that

$$\frac{y_2 - b}{r+\delta} - \frac{\nu(c_a)}{r+\delta} = k$$

For any $\bar{c} \in [0, c_a)$, there exists a $q_c(\bar{c}) < \infty$ such that for any $q^e(\bar{c}) > q_c(\bar{c})$, there is a $q_2^1 < \infty$ solves equation $(Eq-2)$.

Finally, consider equation $(Eq-3)$. The left hand side of the equation increases as q_2^1 increases and the right hand side of the equation decreases as q^e increases. Therefore, equation $(Eq-3)$ characterize q_2^1 an increasing function of q^e. When q_2^1 converges to 0, the left hand side of the equation converges to infinity, and when q_2^1 converges to infinity, it converges to k. Define q_d such that

$$\frac{r+\delta}{r+\delta+(1-\delta)f(q_d)}\left[\frac{k}{q_d f(q_d)} - \frac{y_2-y_1}{r+\delta}\right] + \frac{k}{q_d f(q_d)}\left(\frac{1-\eta(q_d)}{\eta(q_d)}\right) = k \tag{20}$$

and $q_2^1 < \infty$ if and only if $q^e < q_d$. It is easy to see that $q_c < q_b$ for any $k > 0$ by comparing equation (20) and the equation in Lemma 1 that characterize q_b.

Given the monotonic property between q_2^1 and q^e, characterized by equation $(Eq-2)$ and $(Eq-3)$, a unique solution exists if and only if $q_c(\bar{c}) < q_d$. Define c_b such that $q_c(c_b) = q_d$. Notice that $c_a > c_b$ since the left hand side of equation (20) converges to $k - (y_2 - y_2)/(r+\delta) < 0$ as q_d converges to infinity. Therefore, $q_c(c_b) = q_d < q_c(c_a)$ and $c_b < c_a$. For any $\bar{c} \in [0, c_b)$, a unique solution exists to the problem and $0 < q_2^1 < \infty$, $q^e \in (q_b(\bar{c}), q_c)$ and $0 < q_2^2 < \infty$. *QED*

Construct $D(\bar{c}) = V_2^2 - V_2^1$, using the value of V_2^1 and V_2^1 implied by the q_2^2, q_2^1 and q^e as a function of $\bar{c}$. From equation (16), $D(\bar{c})$ can be expressed as

$$D(\bar{c}) = \frac{1-\eta(q_2^2(\bar{c}))}{\eta(q_2^2(\bar{c}))}\frac{k}{q_2^2(\bar{c})} - \lambda q_2^1\left[\frac{1-\eta(q_2^1(\bar{c}))}{\eta(q_2^1(\bar{c}))}\right]\frac{k}{q_2^1(\bar{c})} \tag{21}$$

To show that fixed point exists for $\bar{c} \in [0, c_b)$, I first show that $D(\bar{c})$ is an increasing function of

$\bar{c}$. I then show that $D(c_b) < c_b$ while $D(0) > 0$. Together, a fixed point $D(\bar{c}) = \bar{c}$ exists for $\bar{c} \in [0, c_b)$.

Consider first the monotonicity of $D(\bar{c})$. From equation $(Eq-1)$, q_2^2 increases as $\bar{c}$ increases. The first item in $D(\bar{c})$ is an increasing function of $\bar{c}$. From equation $(Eq-2)$, for any given q^e, an increase of $\bar{c}$ increases q_2^1. Combining with equation $(Eq-3)$, q_2^1 and q^e increases as $\bar{c}$ increases. Given that λq_2^1 is a decreasing function of q^e, the second term after the minus sign decreases as $\bar{c}$ increases. Together, $D(\bar{c})$ is an increasing function of $\bar{c}$.

Now consider the upper bound at $\bar{c} = c_b$. When $\bar{c} = c_b$, $q_c(c_b) = q_d$ and $q_2^1(c_b) = \infty$. Equation $(Eq-2)$ becomes:

$$\frac{y_2 - b}{r+\delta} - \frac{\nu(c_b)}{r+\delta} = \frac{k}{q_d f(q_d)\eta(q_d)} \tag{22}$$

where q_d is defined by equation (20). Substitute the above equation into equation $(Eq-1)$

$$D(c_b) - c_b = \frac{k(r+\delta)}{q_d f(q_d)\eta(q_d)} - \frac{k(r+\delta)}{q_2^2(\bar{c}) f(q_2^2(\bar{c}))\eta(q_2^2(\bar{c}))}$$

$D(c_b) - c_b < 0$ if and only if $q_2^2(\bar{c}) < q_d$. Constructing the following using equation $(Eq-1)$ and (22)

$$\frac{k(r+\delta)}{q_2^2 f(q_2^2(\bar{c}))\eta(q_2^2(\bar{c}))} + \frac{1-\eta(q_2^2(\bar{c}))}{\eta(q_2^2(\bar{c}))}\frac{k}{q_2^2(\bar{c})} - \frac{k(r+\delta)}{q_d f(q_d)\eta(q_d)} - \frac{1-\eta(q_d)}{\eta(q_d)}\frac{k}{q_d} = c_b - \frac{1-\eta(q_d)}{\eta(q_d)}\frac{k}{q_d}$$

Given that $\dfrac{k}{qf(q)\eta(q)} + \dfrac{1-\eta(q)}{\eta(q)}\dfrac{k}{(r+\delta)q}$ is a decreasing function of q, $q_2^2(\bar{c}) < q_d$ would imply that

$$c_b > \left(\frac{1}{\eta(q_d)} - 1\right)\frac{k}{q_d} \tag{23}$$

Lemma 4 *Assume* $(y_1 - b) > \dfrac{\delta}{1+r}(y_2 - y_1)$. *There exists a number* $\bar{k}_1 > 0$ *such that for* $k \in (0, \bar{k}_1]$, $D(c_b) < c_b$.

Proof: Differentiate equation (20), it is easy to verify that $\dfrac{dq_d}{q_k} > 0$. and $\lim_{k\to 0} q_d = 0$. Also notice that

$$\lim_{k\to 0}\frac{k}{q_d f(q_d)} = \lim_{k\to 0}\frac{k}{q_d} = 0$$

and

$$\lim_{k\to 0}\frac{k}{q_d f(q_d)\eta(q_d)} = \frac{y_2 - y_1}{1+r}$$

Since $c_b = \nu(c_b) + F(c_b)/\theta > \nu(c_b)$. A sufficient condition for equation (23) to hold is that

$$\nu(c_b) - \left(\frac{1}{\eta(q_d)} - 1\right)\frac{k}{q_d} = y_2 - b - \frac{k(r+\delta+f(q_c))}{q_d f(q_d)\eta(q_d)} + \frac{k}{q_c} > 0$$

In the limit at $k \to 0$

$$\lim_{k\to 0}\left[\nu(c_b) - \left(\frac{1}{\eta(q_d)} - 1\right)\frac{k}{q_d}\right] = y_1 - b - \frac{\delta}{1+r}(y_2 - y_1)$$

which is positive if $y_1 - b > \frac{\delta}{1+r}(y_2 - y_1)$, as assumed. Therefore, by continuity, there is a $\bar{k}_1 > 0$ such that $D(c_b) - c_b < 0$. *QED*

Finally, consider the lower boundary at $\bar{c} = 0$. Denote $q^1 = q_2^1(0)$, $q^2 = q_2^2(0)$ and $q_0^e = q^e(0)$. Rewrite equation $(Eq-1)$ and $(Eq-2)$ as

$$\frac{y_2 - b}{r+\delta} - \frac{r}{r+\delta}\frac{V(\{(b,0),2\})}{1+r} = \frac{k}{q^2 f(q^2)\eta(q^2)} \tag{24}$$

and

$$\frac{y_2 - b}{r+\delta} - \frac{k}{q_0^e f(q_0^e)\eta(q_0^e)} - \frac{r}{r+\delta}\frac{V(\{(b,0),2\})}{1+r} = \lambda q_2^1\left(\frac{1-\eta(q^1)}{\eta(q^1)}\right)\frac{k}{q^1 f(q^1)} \tag{25}$$

First notice that $\lambda q^1 > 1$. From equation (11), $\lambda q^1 > 1$ if and only if $d\hat{S}(q)/dq > 0$, which holds for $q < q_b$ as shown in Lemma 2. Combining equation (24) and (25)

$$\frac{k}{q_0^e f(q_0^e)\eta(q_0^e)} - \frac{k_2}{q^2 f(q^2)} = \left(\frac{1-\eta(q^2)}{\eta(q^2)}\right)\frac{k}{q^2 f(q^2)} - \lambda q^1\left(\frac{1-\eta(q^1)}{\eta(q^1)}\right)\frac{k}{q^1 f(q^1)} \tag{26}$$

substitute equation $(Eq-3)$

$$\frac{k}{q^1 f(q^1)} - \frac{k}{q^2 f(q^2)} + \frac{r+\delta}{[r+\delta+(1-\delta)f(q_0^e)]}\left[\frac{y_2 - y_1}{r+\delta} - \frac{k}{q_0^e f(q_0^e)}\right] + \frac{k}{q_0^e f(q_0^e)}$$
$$= \left(\frac{1-\eta(q^2)}{\eta(q^2)}\right)\frac{k}{q^2 f(q^2)} - \lambda q^1\left(\frac{1-\eta(q^1)}{\eta(q^1)}\right)\frac{k}{q^1 f(q^1)}$$

Given that $\lambda q_2^1 > 1$ and that the second term is greater than zero, the following inequality must hold

$$\frac{k}{q^1 f(q^1)} - \frac{k}{q^2 f(q^2)} < \left(\frac{1-\eta(q^2)}{\eta(q^2)}\right)\frac{k}{q^2 f(q^2)} - \left(\frac{1-\eta(q^1)}{\eta(q^1)}\right)\frac{k}{q^1 f(q^1)}$$

Since both $\frac{k}{qf(q)}$ and $\left(\frac{1-\eta(q)}{\eta(q)}\right)\frac{k}{qf(q)}$ are decreasing functions of q, it must be that $q^1 > q^2$.

Rewrite equation (26) as

$$\frac{k}{q_0^e f(q_0^e)\eta(q_0^e)} - \frac{k}{q^2 f(q^2)} = \frac{1}{f(q^2)}\left[\left(\frac{1-\eta(q^2)}{\eta(q^2)}\right)\frac{k}{q^2} - \lambda q^1\left(\frac{1-\eta(q^1)}{\eta(q^1)}\right)\frac{k}{q^1}\frac{f(q^2)}{f(q^1)}\right] = \frac{1}{f(q^2)}D(0)$$

and $D(0) > 0$ if $\dfrac{k}{q_0^e f(q_0^e)\eta(q_0^e)} > \dfrac{k}{q^2 f(q^2)}$.

Using equation $(Eq-1)$ evaluated at $\bar{c}=0$. Since the right hand side of the equation decreases with q^2, $dq^2/dk > 0$ and it is easy to show that $\lim_{k\to 0} q_2^2 = \lim_{k\to 0} k/q^2 f(q^2) = 0$.

As showed in the last lemma that $\lim_{k\to 0} \dfrac{k}{q_c f(q_c)\eta(q_c)} = \dfrac{y_2 - y_1}{1+r}$. Also given that $q^e(\bar{c})$ is an increasing function of $\bar{c}$, therefore

$$\lim_{k\to 0} \frac{k}{q_0^e f(q_0^e)\eta(q_0^e)} > \lim_{k\to 0} \frac{k}{q_c f(q_c)\eta(q_c)} = \frac{y_2 - y_1}{1+r}$$

As a result, there exists an $\bar{k}_2 > 0$ such that for any $k \in (0, \bar{k}_2]$, $D(0) > 0$.

Overall, assuming $(y_1 - b)/(y_2 - y_1) > \delta/(1+r)$, there is a number $\bar{k} > 0$ such that for any $k \in (0, \bar{k}]$, there exists a unique set of $\{q_{12}^*, q_{22}^*, q_e^*, \omega_{12}^*, \omega_{22}^*, \omega_e^*\}$ that solves the problems $(\mathbf{P-1})$ to $(\mathbf{P-3})$. The solution must be one of the two cases: if the solution to equations $(Eq-1)$ to $(Eq-3)$ has $q^e \geq q_a$, an interior solution solve the problems and is characterized by equation $(Eq-1)$ to $(Eq-3)$, together with equations (17) to (19). Otherwise, a corner solves the problem where $q_e^* = q_a$ while q_{12}^* and q_{22}^* are characterized by equations $(Eq-1)$ and $(Eq-2)$ evaluated at $q^e = q_a$, together with equations (17) to (19).

High School workers

Consider the optimal search problem $(\mathbf{P-4})$ of a highs school worker. Ignore the incentive compatibility constraint, the first order conditions for an interior solution are given by

$$\lambda q = 1$$

and

$$\frac{V(\{(\omega,1),1\})}{1+r} - \frac{V(\{(b,0),1\})}{1+r} = \lambda q \frac{1-\eta(q)}{\eta(q)} \frac{y_1 - \omega}{r+\delta}$$

Substituting $V(\{(\omega,1),1\})$ and the zero profit condition gives

$$\frac{y_1}{r+\delta} - \frac{r}{r+\delta}\frac{V(\{(b,0),1\})}{1+r} = \frac{k}{qf(q)\eta(q)} \tag{27}$$

Substitute $V(\{(b,0),1\})$ from the above back to the first order condition, an interior solution to the problem must satisfies

$$y_1 - b = \frac{k}{qf(q)}\left[r+\delta+\left(\frac{1-\eta(q)}{\eta(q)}\right)[r+\delta+f(q)]\right] \tag{28}$$

The right hand side is a decreasing function of q, it converges to zero when q converges to infinity and it converges to $k(r+\delta)$, there must exist a $q < \infty$ that solves the problem. The wage

ω follows

$$\omega = y_1 - \frac{(r+\delta)k}{qf(q)} \tag{29}$$

If the Incentive Compatibility Constraint is binding, then equilibrium allocation of high school workers is such that the college workers are indifferent between mimicking the high school workers versus the optimal allocation $(\omega_{12}^*, q_{12}^*)$ when applying for routine jobs. Let (q_1, ω_1) be the allocation such that the college worker is indifferent,

$$V_2^1 = f(q_1)\left[\frac{V(\{(\omega_1, 1), 2\})}{1+r} - \frac{V(\{(b,0),2\})}{1+r}\right] + \frac{V(\{(b,0),2\})}{1+r} \tag{30}$$

with the zero profit condition

$$\omega_1 = y_1 - \frac{(r+\delta)k}{q_1 f(q_1)} \tag{31}$$

Denote $\{q_1^*, \omega_1^*\}$ the solution to problem $(\mathbf{P-4})$, which must be one of the two cases. If the incentive compatibility constraint is not binding, the solution to problem $(\mathbf{P-4})$ is interior and is characterized by equations (38) and (36). Otherwise, the solution to the problem is constrained and characterized by equations (30) and (31).

It is now straight forward to characterize the distribution of workers ψ.

$$\psi(\{(b,0),2\}) = \frac{\delta}{\delta + (1-F(c_0))f(q_{12}^*) + F(c_0)f(q_{22}^*)} * \phi$$

$$\psi(\{(\omega_{22}^*, 2), 2\}) = \frac{F(c_0)f(q_{22}^*)}{\delta}\psi(\{(b,0),2\})$$

$$\psi(\{(\omega_{12}^*, 1), 2\}) = \frac{(1-F(c_0))f(q_{12}^*)}{\delta + (1-\delta)f(q_e^*)}\psi(\{(b,0),2\})$$

$$\psi(\{(\omega_e^*, 2), 2\}) = \frac{(1-\delta)f(q_e^*)}{\delta}\psi(\{(\omega_{12}^*, 1), 2\})$$

$$\psi(\{(b,0),1\}) = \frac{\delta}{\delta + f(q_1^*)} * (1-\phi)$$

$$\psi(\{(\omega_1^*, 1), 1\}) = (1-\phi) - \psi(\{(b,0),1\})$$

This concludes the proof of **Proposition 2**. **QED**

Proof of Proposition 3

I show a sufficient condition for the incentive compatibility constrain to bind by constructing an example at the limit. First, take $\theta \to 0$, the average cost of applying for cognitive job goes to infinity. In the limit, all college workers apply for routine jobs only when unemployed, and search on-the-job once employed.

Let the cost of posting a poaching offer to be k_p, while the cost of posting when hiring unemployed workers remains to be k. Take $k_p \to 0$ such that poaching employed workers is free. The job arrival rate of on-the-job search $f(q^e)$ converges to 1, and the poaching wage ω^e converges to the productivity y_2. Once matched with a routine job, the worker stays only for one period and switches employer and receives y_2 in the next period. Both the quite rate and the poaching wage are independent of the current wage.

The value of employment in a routine job with wage ω for college worker is:

$$V(\{(\omega,1),2\}) = \omega + \frac{\delta V(\{(b,0),2\})}{1+r} + (1-\delta)\frac{V(\{(y_2,2),2\})}{1+r}$$

substituting $V(\{(y_2,2),2\}$ gives

$$\frac{V(\{(\omega,1),2\})}{1+r} = \frac{\omega}{1+r} + \frac{(1-\delta)}{(r+\delta)(1+r)}y_2 + \frac{\delta}{r+\delta}\frac{V(\{(b,0),2\})}{1+r}$$

Consider now the optimal search problem of an unemployed college worker when applying for a routine job

$$\max_{q,\omega} f(q)\frac{V(\{(\omega,1),2\})}{1+r} + [1-f(q)]\frac{V(\{(b,0),2\})}{1+r}$$

s.t.

$$-k + qf(q)\frac{y_1-\omega}{1+r} = 0$$

Note that the discount rate of firms is $(1+r)$ given that the quit rate is $1-\delta$. The corresponding first order conditions are:

$$\lambda q = 1$$

where λ is the Lagrange multiplier

$$\frac{V(\{(\omega,1),2\})}{1+r} - \frac{V(\{(b,0),2\})}{1+r} = \lambda q \frac{1-\eta(q)}{\eta(q)}\frac{y_1-\omega}{1+r}$$

Combining the first order conditions and substituting the zero profit condition

$$\frac{V(\{(\omega,1),2\})}{1+r}-\frac{V(\{(b,0),2\})}{1+r}=\frac{1-\eta(q)}{\eta(q)}\frac{k}{qf(q)}$$

Let (q_2,ω_2) denote the solution to the problem, q_2 and ω_2 satisfy

$$\frac{y_1}{1+r}+\frac{(1-\delta)y_2}{(1+r)(r+\delta)}-\frac{r}{r+\delta}\frac{V(\{(b,0),2\})}{1+r}=\frac{k}{q_2f(q_2)\eta(q_2)} \tag{32}$$

and

$$\omega_2=y_1-\frac{k(1+r)}{q_2f(q_2)} \tag{33}$$

The value of unemployment for college workers $V(\{(b,0),2\}$ equals

$$V(\{(b,0),2\})=b+f(q_2)\left[\frac{V(\{(\omega_2,1),2\})}{1+r}-\frac{V(\{(b,0),2\})}{1+r}\right]+\frac{V(\{(b,0),2\}}{1+r}$$

substituting the first order conditions such that

$$\frac{r}{1+r}V(\{(b,0),2\})=b+\frac{1-\eta(q_2)}{\eta(q_2)}\frac{k}{q_2}$$

substituting this back to (32), q_2 must solve

$$\frac{y_1}{1+r}+\frac{(1-\delta)y_2}{(1+r)(r+\delta)}=\frac{1}{r+\delta}\left(b+\frac{1-\eta(q_2)}{\eta(q_2)}\frac{k}{q_2}\right)+\frac{k}{q_2f(q_2)\eta(q_2)} \tag{34}$$

Similarity, consider now the unconstrained optimal search problem of an unemployed high school worker

$$\max_{q,\omega} f(q)\frac{V(\{(\omega,1),1\})}{1+r}+[1-f(q)]\frac{V(\{(b,0),1\})}{1+r}$$

s.t.

$$-k+qf(q)\frac{y_1-\omega}{r+\delta}=0$$

The corresponding first order conditions together imply

$$\frac{V(\{(\omega,1),1\})}{1+r}-\frac{V(\{(b,0),1\})}{1+r}=\frac{1-\eta(q)}{\eta(q)}\frac{k}{qf(q)}$$

Let (q_1,ω_1) denote the solution to the above problem, q_1 and ω_1 satisfy

$$\frac{y_1}{r+\delta}-\frac{r}{r+\delta}\frac{V(\{(b,0),1\})}{1+r}=\frac{k}{q_1f(q_1)\eta(q_1)} \tag{35}$$

and

$$\omega_1=y_1-\frac{k(r+\delta)}{q_1f(q_1)} \tag{36}$$

The return to search from unemployment to high school workers U_1 follows

$$V(\{(b,0),1\}) = b + f(q_1)\left[\frac{(V(\{(\omega_1,1),1\}))}{1+r} - \frac{V(\{(b,0),1\})}{1+r}\right] + \frac{U_1}{1+r}$$

substitute the FOC, one obtains

$$\frac{r}{1+r}V(\{(b,0),1\}) = b + \frac{1-\eta(q_1)}{\eta(q_1)}\frac{k}{q_1} \tag{37}$$

and q_1 solves

$$\frac{y_1}{r+\delta} = \frac{1}{r+\delta}\left(b + \frac{1-\eta(q_1)}{\eta(q_1)}\frac{k}{q_1}\right) + \frac{k}{q_1 f(q_1)\eta(q_1)} \tag{38}$$

Compare equation (34) and (38), when $y_2 = y_1$, the left hand sides of the two equations are equals, which implies $q_1 = q_2$. Also compare equations (33) and (36), when $q_1 = q_2$, $\omega_1 > \omega_2$. Since $V(\{(\omega,1),2\})$ increases with ω. college workers would be strictly better off applying to $\{\omega_1, q_1\}$ and search on-the-job, which means

$$f(q_2)\left[\frac{V(\{(\omega_2,1),2\})}{1+r} - \frac{V(\{(b,0),2\})}{1+r}\right] < f(q_1)\left[\frac{V(\{(\omega_1,1),2\})}{1+r} - \frac{V(\{(b,0),2\})}{1+r}\right]$$

Notice that when $q_1 = q_2$, $V(\{(b,0),1\}) = V(\{(b,0),2\})$ and $V(\{(\omega_2,1),2\}) = V(\{(\omega_1,1),1\})$. When $y_1 = y_2$, the total surplus of a match, the workers' share of surplus are equal for both types of workers, even though the wage received out of unemployment are not the same.

So far I have shown that at the limit when $\theta = 0$, $k_p = 0$ and $y_2 = y_1$, college workers have strict incentive to mimic high school workers. Fixing y_1, since the problem is continuous in y_2 (see equation (34)), there exists a level $\bar{y} > y_1$ such that for all $y_2 \in (y_1, \bar{y})$, the incentive compatibility constraint binds.

Next I show that the problem is also continuous in θ. Since θ enters the problem of college workers through the the cut-off cost $\bar{c}$ with $\nu(\bar{c}) = \bar{c} - F(\bar{c})/\theta$, the problem is continuous in θ. Therefore there exist a $\bar{\theta}$ such that at $k_p = 0$, the incentive compatibility constraint binds when $y_2 \in (y_1, \bar{y})$ and $\theta \in (0, \bar{\theta})$. Finally, I show that the problem is also continuous in k_p. Rewrite the condition in Lemma one in k_p:

$$q^e f(q^e)\left(\frac{y_2 - \omega^e}{r+\delta}\right) = k_p$$

$$\frac{\omega^e - \omega_2}{r+\delta+(1-\delta)f(q^e)} = \frac{1-\eta(q)}{\eta(q)}\frac{k_p}{q^e f(q^e)}$$

where ω_2 is the wage of an college worker employed in routine jobs and ω^e is her wage the after on-the-job and q^e if associated queue. It is clear that both q^e and ω^e are continuous in k_p. As $k_p \to 0$, $q^e \to 0$. Since $\omega^e - \omega_2$ is greater than 0, therefore $k_p/q^e f(q^e) \to 0$ and $w^e \to y_2$.

There exist numbers $\bar{k}_1 > 0$, $\bar{\theta} > 0$ and $\bar{y} > y_1$ such that for all $k \in (0, \bar{k}_1)$, $\theta \in (0, \bar{\lambda})$ and $y_2 \in (y_1, \bar{y})$, the incentive compatibility constraint is binding.

Now I show that in the limit case when the incentive compatibility constraint is binding, the equilibrium unemployment rate of high school workers is inefficiently high. Denote (ω_1^*, q_1^*) the equilibrium allocation of high school workers. The allocation is such that college workers are indifferent between applying to (ω_2, q_2) and (ω_1^*, q_1^*), i.e.

$$f(q_2)\left[\frac{V(\{(\omega_2,1),2\})}{1+r} - \frac{V(\{(b,0),2\})}{1+r}\right] = f(q_1^*)\left[\frac{V(\{(\omega_1^*,1),2\})}{1+r} - \frac{V(\{(b,0),2\})}{1+r}\right] \tag{39}$$

The allocation also generate zero expected profits to routine employers anticipating high school workers, such that

$$\omega_1^* = y_1 - \frac{k(r+\delta)}{q_1^* f(q_1^*)}$$

The above two equations have two potential solutions; one has a larger queue than q_1 while the other has a smaller queue. To see which possible solution is the equilibrium allocation, compare the slopes of indifference curves between high school and college workers. The indifference curves have positive slopes in the wage-queue space, since workers prefer both a higher wage and a lower queue. If the indifferent curve of college workers have a steeper slope in $d\omega/dq$, then high school workers would strictly prefer the allocation with a higher queue.

Consider the slope of indifference curve of high school workers, evaluated at (ω_1^*, q_1^*)

$$\left(\frac{d\omega}{dq}\right)^1 |_{(\omega_1^*, q_1^*)} = -f'(q_1^*)(r+\delta)\left[\frac{V(\{(\omega_1^*,1),1\})}{1+r} - \frac{V(\{((b,0),1)\})}{1+r}\right]$$

the slope of indifference curve of college workers, evaluated at (ω_1^*, q_1^*). Notice that in the limit case, on-the-job decision does not depends on the current wage

$$\left(\frac{d\omega}{dq}\right)^2 |_{(\omega_1^*, q_1^*)} = -f'(q_1^*)(r+\delta)\left[\frac{V(\{(\omega_1^*,1),2\})}{1+r} - \frac{V(\{((b,0),2)\})}{1+r}\right]$$

In the limit when $\theta = 0$, $k_p = 0$ and $y_2 = y_1$, $V(\{(b,0),2\}) = V(\{(b,0),1\})$, while $V(\{(\omega_1^*,1),2\}) > V(\{(\omega_1^*,1),1\})$, as discussed earlier. The slope of indifference curve of a college worker is higher than of a high school worker. This means comparing with college workers, high school workers would prefer a larger increase in queue for a marginal increase in wage. For two allocations that college workers are indifferent (both satisfy equation (39)), high school worker would strictly prefer the one with a large queue. Therefore, $q_1^* > q_1$.

Since the unemployment rate of high school workers equals to $u_1^* = \delta/[\delta + f(q_1^*)]$, $u_1^* > u_1 =$

$\delta/[\delta + f(q_1)]$. The equilibrium unemployment rate of high school worker is higher than the constrained efficient level.

As showed earlier, the problem is continuous in θ, k_p and y_2, therefore there exits some cutoffs $\bar{k}_1 > 0$, $\bar{\theta} > 0$ and $\bar{y}$ such that the incentive compatibility constraint is binding and the unemployment rate of high school workers is inefficiently high. *QED*

Proof of Proposition 4

Given that the economy suffers from adverse selection, the return to search of unemployed workers with vocational education is greater than the return to search of high school workers, $U_v - U_h > 0$. From equations (9) and (10), the cutoffs of post-secondary education are $\bar{e} = (U_v - U_h)/\beta$ and $\underline{e} = (U_c - U_v)/(\alpha - \beta)$.

Denote $\bar{\beta}$ such that $\bar{e} = \underline{e}$. When the cost rate of vocational equals to $\bar{\beta}$, an individual with cost of education greater than $\bar{e}$ does not pursuit post-secondary education. An individual with cost less or equal to $\bar{e}$ weakly prefers college education. $\bar{\beta}$ solves:

$$\bar{\beta} = \alpha \frac{U_v - U_h}{U_c - U_h} \tag{40}$$

Given that $U_v > U_h$ and $U_c > U_h$, $\bar{\beta} > 0$ for any $\alpha > 0$. The demand of vocational education is positive when $\beta < \bar{\beta}$. *QED*

Proof of Proposition 5

Assume that the economy suffers from adverse selection. Fix α. Consider the cutoffs as function of β. Let $\bar{e}(\beta) = (U_v - U_h)/\beta$. $\bar{e}(\beta)$ decreases with β such that $\bar{e}'(\beta) < 0$. Similarly, let $\underline{e}(\beta) = (U_c - U_v)/(\alpha - \beta)$ and $\underline{e}(\beta)$ increases with β such that $\bar{e}'(\beta) > 0$. Also consider the ex-ante utility of a worker U_0 as the function of β such that

$$U_0(\beta) = G(\underline{e}(\beta))U_c + [G(\bar{e}(\beta)) - G(\underline{e}(\beta))]U_v + [1 - G(\bar{e}(\beta))]U_h - \beta\mathbb{E}[e|\underline{e}(\beta) \le e < \bar{e}(\beta)] - \alpha\mathbb{E}[e|e \le \underline{e}(\beta)]$$

Differentiate $U_0(\beta)$ with respect to β

$$\frac{dU_0(\beta)}{d\beta} = -\int_{\underline{e}(\beta)}^{\bar{e}(\beta)} eg(e)de \tag{41}$$

where $g(\cdot)$ is the density function of G.

I now consider the ex-ante utility in an economy where the educational cost to vocational education is $\bar{\beta}$, where $\bar{\beta}$ is defined in Proposition 4. When $\beta = \bar{\beta}$, no individual takes vocational education because the cost is too high. $U_0(\bar{\beta})$ equals the level of ex-ante utility if workers can choose either high school or college with cost rate α. Notice that $\bar{e}(\bar{\beta}) = \underline{e}(\bar{\beta})$, so $\left.\frac{dU_0(\beta)}{d\beta}\right|_{\beta=\bar{\beta}} = 0$.

For any $\beta < \bar{\beta}$, $\bar{e}(\beta) > \underline{e}(\beta)$. Therefore $\frac{dU_0(\beta)}{d\beta} < 0$ for $\beta \in (0, \bar{\beta})$. By continuity, $U_0(\bar{\beta}) < U_0(\beta)$ for any $\beta \in (0, \bar{\beta})$. The ex-ante utility of a worker is higher in economy with positive demand of vocational education than in the economy when no workers can take vocational education. *QED*

References

Acemoglu, D. and Autor, D. (2011). Skills, tasks and technologies: Implications for employment and earnings. *Handbook of labor economics*, 4:1043–1171.

Albrecht, J. and Vroman, S. (2002). A matching model with endogenous skill requirements. *International Economic Review*, 43(1):283–305.

Autor, D. et al. (2010). The polarization of job opportunities in the us labor market: Implications for employment and earnings. *Center for American Progress and The Hamilton Project*.

Autor, D. H., Dorn, D., Hanson, G. H., and Song, J. (2014). Trade adjustment: Worker-level evidence. *The Quarterly Journal of Economics*, 129(4):1799–1860.

Autor, D. H., Levy, F., and Murnane, R. J. (2003). The skill content of recent technological change: An empirical exploration. *The Quarterly Journal of Economics*, 118(4):1279–1333.

Barnichon, R. and Zylberberg, Y. (2014). Under-employment and the trickle-down of unemployment.

Beaudry, P., Green, D. A., and Sand, B. M. (2013). The great reversal in the demand for skill and cognitive tasks. Technical report, National Bureau of Economic Research.

Chen, Y., Doyle, M., and Gonzalez, F. M. (2017). Mismatch as choice. *Working Paper*.

Cho, I.-K. and Kreps, D. M. (1987). Signaling games and stable equilibria. *The Quarterly Journal of Economics*, 102(2):179–221.

Clark, B., Joubert, C., and Maurel, A. (2014). The career prospects of overeducated americans. Technical report, National Bureau of Economic Research.

Cooney, R. (2013). Occupational licensing in intermediate-skill occupations: The case of drivers in the land transport industry. *Journal of industrial relations*, 55(5):743–759.

Delacroix, A. and Shi, S. (2006). Directed search on the job and the wage ladder. *International Economic Review*, 47(2):651–699.

Dolado, J. J., Jansen, M., and Jimeno, J. F. (2009). On-the-job search in a matching model with heterogeneous jobs and workers. *The Economic Journal*, 119(534):200–228.

Gautier, P. A. (2002). Unemployment and search externalities in a model with heterogeneous jobs and workers. *Economica*, 69(273):21–40.

Gautier, P. A., Van den Berg, G. J., Van Ours, J. C., and Ridder, G. (2002). Worker turnover at the firm level and crowding out of lower educated workers. *European Economic Review*, 46(3):523–538.

Goos, M., Manning, A., and Salomons, A. (2014). Explaining job polarization: Routine-biased technological change and offshoring. *The American Economic Review*, 104(8):2509–2526.

Guerrieri, V., Shimer, R., and Wright, R. (2010). Adverse selection in competitive search equilibrium. *Econometrica*, 78(6):1823–1862.

Kleiner, M. M. (2005). Reforming occupational licensing policies. *The Hamilton Project*.

Kleiner, M. M. and Krueger, A. B. (2013). Analyzing the extent and influence of occupational licensing on the labor market. *Journal of Labor Economics*, 31(2 pt 2).

Moen, E. R. (1997). Competitive search equilibrium. *Journal of political Economy*, 105(2):385–411.

Sahin, A., Song, J., and Hobijn, B. (2010). The unemployment gender gap during the 2007 recession. *Current Issues in Economics and Finance*, 16(2).

Van den Berg, G. J., Holm, A., and Van Ours, J. C. (2002). Do stepping-stone jobs exist? early career paths in the medical profession. *Journal of Population Economics*, 15(4):647–665.

Vedder, R., Denhart, C., and Robe, J. (2013). Why are recent college graduates underemployed? university enrollments and labor-market realities. *Center for College Affordability and Productivity (NJ1)*.